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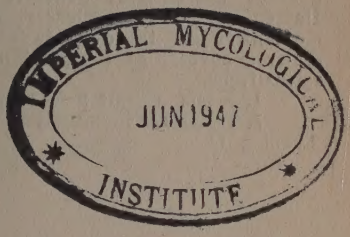

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MATHÉMATIQUES

LA GÉOMETRIE D'UN CHAMP DE VECTEURS

Par S. S. BUSCHEGUENNCE

(Présenté par N. N. Lusin, de L'Académie, le 17. VII. 1944)

Depuis longtemps plusieurs géomètres (Lie, Lilienthal, Voss, Rogers, Darboux, Sintsov, Inzigers, Knoll, *etc.*) ont fixé leur attention sur les courbes vérifiant l'équation

$$A(x, y, z)\,dx + B(x, y, z)\,dy + C(x, y, z)\,dz = 0$$

ou, ce qui est le même, sur les courbes orthogonales aux courbes d'une congruence:

$$\frac{dx}{A} = \frac{dy}{B} = \frac{dz}{C}$$

Pour ce champ de vecteurs $(A : B : C)$ on a déterminé les lignes de courbure, les directions conjuguées, les directions asymptotiques, les lignes géodésiques et on a trouvé une série de propriétés. Ma Note a pour but de traiter le problème du champ par la méthode de Cartan de repère mobile, à compléter les définitions des conceptions fondamentales, et enfin, à résoudre une suite de questions sur la structure du champ.

1. Soit (J_1, J_2, J_3) un repère mobile à trois orthes, lié à un point $\overline{M}$; pour un déplacement d'un point $\overline{M}$ nous posons

$$d\overline{M} = \omega_0^\alpha J_\alpha \tag{1}$$

Soit ensuite

$$\overline{\omega} = pJ_1 + qJ_2 + rJ_3 \tag{2}$$

le vecteur de Darboux, c'est-à-dire le vecteur de la rotation (instantanée) d'un repère pour un déplacement choisi $d\overline{M}$; alors

$$dJ_1 = rJ_2 - qJ_3;\ dJ_2 = pJ_3 - rJ_1;\ dJ_3 = qJ_1 - pJ_2 \tag{3}$$

Les formes de Pfaff introduites sont liées par les équations de structure:

$$\left.\begin{aligned} (\omega_0^1)' &= r\omega_0^2 - q\omega_0^3, & p' &= rq \\ (\omega_0^2)' &= p\omega_0^3 - r\omega_0^1, & q' &= pr \\ (\omega_0^3)' &= q\omega_0^1 - p\omega_0^2, & r' &= qp \end{aligned}\right\} \tag{4}$$

Pour simplifier je désignerai le produit extérieur des formes sans aucun indice comme le produit ordinaire. Nous supposons toujours que l'on a $\omega_0^1\omega_0^2\omega_0^3 \neq 0$ et posons

$$p = p_\alpha\omega_0^\alpha,\ q = q_\alpha\omega_0^\alpha,\ r = r_\alpha\omega_0^\alpha \tag{5}$$

Le vecteur de Darboux $\overline{\omega}$ est lié au repère choisi, mais la projection de $\overline{\omega}$ sur le plan perpendiculaire à J_3 ne dépend pas de la rotation du repère autour de l'axe J_3. Par conséquent, la direction normale au vecteur J_3 et au vecteur de Darboux est indépendante du choix du couple (J_1, J_2).

2. Un déplacement orthogonal au vecteur J_3 du champ doit vérifier l'équation

$$J_3 d\overline{M} = 0 \tag{6}$$

ou bien

$$\omega_0^3 = 0 \tag{6'}$$

La dernière donne dans le cas général les courbes orthogonales au vecteurs du champ, ou bien, tout court, des trajectoires orthogonales du champ. Si l'équation (6′) est complètement intégrable, on a

$$(\omega_0^3)' \omega_0^3 = 0 \tag{7}$$

et nous aurons une famille de surfaces orthogonales aux vecteurs du champ. En développant la condition (7) à l'aide des formules (4), on obtient

$$p\omega_0^2\omega_0^3 + q\omega_0^3\omega_0^1 = 0 \tag{7'}$$

ou bien

$$p_1 + q_2 = 0 \tag{7''}$$

Telle est donc la condition nécessaire et suffisante pour l'existence d'une famille de surfaces orthogonales d'un champ de vecteurs donné.

3. Je nomme ligne de courbure d'un champ une ligne, qui est en chacun de ces points orthogonale simultanément au vecteur correspondant du champ et au vecteur correspondant de Darboux. Les lignes de courbure d'un champ (J_3) doivent vérifier les équations

$$\omega_0^3 = 0, \quad (p)\,\omega_0^1 + (q)\,\omega_0^3 = 0 \tag{8}$$

ou bien les équations

$$\omega_0^3 = 0, \quad p_1 (\omega_0^1)^2 + (p_2 + q_1)(\omega_0^1)(\omega_0^3) + q_2 (\omega_0^2)^2 = 0 \tag{8'}$$

On a ici des produits ordinaires des formes de Pfaff, ce que je désigne en mettant les formes entre parenthèses. Un champ de vecteurs a deux congruences de lignes de courbure. Le long d'une ligne de courbure du champ les vecteurs du champ forment une surface développable, parce que les conditions

$$\omega_0^3 = 0, \quad d\overline{M} J_3\, dJ_3 = 0 \tag{9}$$

sont équivalentes aux équations (8).

Deux lignes de courbure au point $\overline{M}$ ne sont pas orthogonales dans le cas général; la condition (7″) est une condition d'orthogonalité des lignes de courbure. Donc, pour que le champ de vecteurs admette une famille de surfaces orthogonales, il est nécessaire et suffisant que les lignes de courbure du champ soient orthogonales. Si le champ admet une famille de surfaces orthogonales, on voit que les lignes de courbure du champ sont situées sur des surfaces de la famille, et pour celles-ci elles sont des lignes de courbure classiques.

4. Soit la famille (9′) une famille de Lamé, c'est-à-dire, une famille des surfaces appartenant à un système triplement orthogonal; prenons pour (J_1) et (J_2) deux champs de vecteurs tangents aux lignes de courbure des sur-

faces de la famille (6'). Alors les équations des deux familles complémentaires des surfaces seront

$$J_1 d\overline{M} \equiv \omega_0^1 = 0, \quad J_2 d\overline{M} = \omega_0^2 = 0$$

et en exigeant leur intégrabilité complète on obtient la condition

$$r_2 = 0 \tag{10}$$

Donc, pour qu'une famille de surfaces soit une famille de Lamé, il faut et il suffit que le champ des tangents d'une congruence de leurs lignes de courbure admette une famille de surfaces orthogonales.

5. Pour un déplacement quelconque le vecteur

$$\overline{\theta} = pJ_1 + qJ_2 \tag{11}$$

est la projection de vecteur correspondant de Darboux ω sur le plan normal au vecteur J_3. Une ligne orthogonale aux vecteurs du champ (J_3) et le long de laquelle chaque déplacement est parallel au vecteur correspondant $\overline{\theta}$, sera nommée ligne asymptotique du champ. Ainsi les lignes asymptotiques du champ seront définies par les équations:

$$\omega_0^3 = 0, \quad (p)\,\omega_0^1 = (q)\,\omega_0^2 \tag{12}$$

ou bien par les équations:

$$\omega_0^3 = 0, \; q_1 (\omega_0^1)^2 + (q_2 - p_1)(\omega_0^1)(\omega_0^2) - p_2 (\omega_0^2)^2 = 0 \tag{12'}$$

Soit

$$\frac{d^2\overline{M}}{ds^2} = \frac{\overline{\nu}}{\rho}, \quad \left(J_3 \frac{d\overline{M}}{ds} = 0\right)$$

un vecteur de courbure d'une ligne orthogonale aux vecteurs du champ (J_3); alors

$$\frac{1}{R} = \frac{\overline{\nu} J_3}{\rho} = -\frac{d\overline{M}\, dJ_3}{ds^2} = \frac{-(q)\,\omega_0^1 + (p)\,\omega_0^2}{ds^2} \tag{13}$$

d'où il résulte qu'une ligne asymptotique du champ est une ligne dont la binormale coïncide avec le vecteur correspondant du champ en chacun de ses points. Les valeurs extrémales de la courbure normale $\frac{1}{R}$ sont déterminées par l'équation

$$\frac{1}{R} + \frac{q_1 - p_2}{R} - q_1 p_2 - \frac{1}{4}(p_1 - q_2)^2 = 0$$

donc

$$\left.\begin{aligned} M &= \frac{1}{R_1} + \frac{1}{R_2} = p_2 - q_1 \\ K_g &= \frac{1}{R_1 R_2} = p_1 q_2 - p_2 q_1 - \frac{1}{4}(p_1 + q_2)^2 \end{aligned}\right\} \tag{14}$$

Telles sont les expressions de la courbure moyenne et de la courbure gaussienne du champ; si la condition (7'') est remplie, elles se réduisent aux formules classiques pour les surfaces orthogonales du champ. Les directions de courbure normale extrême sont déterminées par l'équation:

$$\frac{p_1 - q_2}{2}(\omega_0^1)^2 + (p_2 + q_1)(\omega_0^1)(\omega_0^2) - \frac{p_1 - q_2}{2}(\omega_0^2)^2 = 0 \tag{15}$$

Ces deux directions sont perpendiculaires, mais dans le cas général elles diffèrent des deux directions des lignes de courbure. Les directions (15) coïncident avec les directions des lignes de courbure seulement dans le cas, où le champ admet une famille de surfaces orthogonales.

En revenant aux lignes asymptotiques, j'affirme que les lignes de courbure du champ sont harmoniquement divisées par les lignes asymptotiques seulement dans le cas, où l'un de ces systèmes forme un système orthogonal.

6. Si la normale principale d'une ligne coïncide en chacun de ces points avec le vecteur correspondant du champ, nous l'appelons ligne géodésique du champ.

Pour le champ (J_3) les équations des géodésiques sont:

$$\omega_0^3 = 0, \ (\omega_0^2)\, d\omega_0^1 - (\omega_0^1)\, d\omega_0^3 - (r)\,\{(\omega_0^2)^2 + (\omega_0^3)^2\} = 0 \tag{16}$$

Soit σ l'angle d'une ligne géodésique avec le vecteur J_1; alors les équations (16) prennent la forme:

$$\omega_0^3 = 0, \quad d\sigma + r = 0 \tag{16'}$$

7. Pour un déplacement quelconque $d\overline{M}$ nous dirons que le déplacement $\delta\overline{M}$ est unilatéralement conjugué à $d\overline{M}$, s'il est orthogonal aux vecteurs consécutifs J_3 et $J_3 + dJ_3$; alors nous avons

$$\omega_0^3(\delta) = 0, \ \{q\,(d)\}\,\omega_0^1(\delta) - \{p\,(d)\}\,\omega_0^2(\delta) = 0 \tag{17}$$

Ces directions ne sont pas symétriques. Elles sont symétriques si la condition (7'') est vérifiée, alors elles sont situées sur les surfaces de la famille orthogonale du champ et sont conjuguées au sens classique.

En suivant une autre voie, on peut définir deux directions conjuguées bilatéralement comme les directions orthogonales au vecteur du champ, qui divisent harmoniquement les directions asymptotiques. Alors nous avons

$$\left.\begin{aligned} q_1\omega_0^1(d)\,\omega_0^1(\delta) + (q_2 - p_1)\,\{\omega_0^1(d)\,\omega_0^2(\delta) + \omega_0^1(d)\,\omega_0^2(d)\} - \\ - p_2\omega_0^2(d)\,\omega_0^2(\delta) = 0 \\ \omega_0^3(d) = 0, \quad \omega_0^3(\delta) = 0 \end{aligned}\right\} \tag{18}$$

9. Soient $d\overline{M}$ et $\delta\overline{M}$ deux déplacements quelconques orthogonales au vecteur du champ (J_3). Le rapport des volumes de deux parallélipipèdes construits sur J_3, $J_3 + dJ_3$, $J_3 + \delta J_3$ et sur J_3, $d\overline{M}$, $\delta\overline{M}$ sera appelé la courbure totale du champ en un point choisi. On a

$$K_t = \frac{J_3(J_3 + dJ_3)\,(J_3 + \delta J_3)}{J_3\, dM\, \delta M} = p_1q_2 - p_2q_1 \tag{19}$$

Si le champ admet une famille orthogonale de surfaces, la courbure totale du champ sera la courbure totale au sens ordinaire pour ces surfaces.

Institut des améliorations agricoles
Williams à Moscou.

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MATHÉMATIQUES

SUR UNE SYNTHÈSE DES DEUX MÉTHODES D'EXPOSER LA THÉORIE DES CHAÎNES DISCRÈTES DE MARKOFF

Par T. A. SARYMSAKOV, membre de l'Académie des Sciences d'Uzbekistan

1. L'ensemble des travaux, consacrés à l'étude des chaînes classiques de Markoff à un ensemble discret d'états possibles, peut être divisé, du moins grossièrement, en deux catégories: les travaux, où l'exposé est de caractère qualitatif, et les travaux, où ce caractère est quantitatif. Les deux systèmes d'exposer ont leurs avantages et leurs défauts. Comme exemple du premier système on peut indiquer l'article de Kolmogoroff [1] et de Doeblin[6]; comme exemple du second système—l'article de Romanovsky [5] dont on peut étendre les résultats principaux au cas des chaînes à une infinité dénombrable de cas possibles par un passage à la limite.

L'avantage du premier système consists en ce que les énoncés des théorèmes prennent une forme qui permet de leur donner une interptétation physique, et son défaut est de ne pas indiquer d'algorithme qui permette de calculer les probabilités. Le second système, au contraire, permet d'écarter les défauts du premier, mais ne possède pas ses advantages.

En qualité d'exemple pour illustrer ces considérations on peut indiquer le théorème IV-a de l'article de Kolmogoroff[1] en le formulant pour le cas des chaînes à un nombre fini d'états possibles et le théorème B de l'article cité de Romanovsky [5] que l'on peut étendre au cas des chaînes à une finité dénombrable d'états possibles; ces deux théorèmes se rapportent à une même question. Dans notre exposé ils sont énoncés sous la forme du théorème 8.

Le but de la présente Note est de donner une synthèse de ces deux systèmes importants pour exposer la théorie des chaînes discrètes de Markoff; sa nécessité résulte des considérations précédentes. L'essai d'une pareille synthèse est partiellement réalisé dans la Note de Doeblin [5]. Pour abréger et simplifier l'exposé nous allons considérer les chaînes de Markoff discrètes homogènes à un nombre fini d'états possibles.

2. La synthèse indiquée est basée sur les théorèmes qui suivent. Les notions que nous utilisons ensuite sans donner leur définitions sont celles de Kolmogoroff[1] et Romanovsky [5].

Indiquons plus précisément la littérature où l'on trouve les principales notions utilisées: matrice stochastique ([5], p. 148), matrice indécomposable et décomposable ([5], p. 153); matrice primitive et imprimitive ([5], p. 160); matrice cyclique d'indice k ([5], p. 162) classe essentielle d'états, ses sous-classes; états non essentiels ([1], 2); chaîne régulière de Markoff, positivement et négativement régulière [4]; principe ergodique [2].

Dans la suite nous désignerons par S l'ensemble de tous les états possibles et par E_i $(i=\overline{1, n})$ les états eux-mêmes.

Les notations M_{ij}, $\Pi_{ij}^{(m)}$ sont dues à Kolmogoroff ([1] p. 2 et 5).

Théorème 1. *Pour que la matrice stochastique* Φ *soit non-décompensable, il faut et il suffit que l'ensemble* S *de tous les états possibles présente une classe essentielle.*

Théorème 2. *Pour que la matrice stochastique* Φ *possède* $\lambda_0 = 1$ *comme racine multiple d'ordre* k, *il faut et il suffit que l'ensemble de tous les états possibles contienne* k *classes essentielles: alors la matrice* Φ *peut être réduite à la forme*

$$\Phi = \left|\begin{array}{cccc} L_{11} & 0 & \dots & 0 \\ 0 & L_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & & \dots & L_{kk} \end{array}\right| \; 0$$

$$S \qquad T$$

où les L_{ii} $(i = T, k)$ *et* T *sont des matrices carrées, et* S *est rectangulaire.*

Théorème 3. *Pour qu'une matrice stochastique indécomposable* Φ *possède pour racines toutes les racines de l'équation*

$$\lambda^k - 1 = 0$$

il faut et il suffit que l'ensemble S *de tous les états possibles présente une classe essentielle formée de* k *sous-classes.*

Théorème 3a. *Pour que l'ensemble de tous les états possibles présente une classe essentielle formée de* k *sous-classes, il faut et il suffit que la matrice stochastique correspondante* Φ *soit une matrice cyclique d'indice* k, *c'est-à-dire qu'elle puisse être réduite à la forme*

$$\Phi = \begin{pmatrix} 0 & L_{12} & 0 & \dots & 0 \\ 0 & 0 & L_{23} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & L_{k-1k} \\ L_{k1} & 0 & 0 & \dots & 0 \end{pmatrix}$$

où les L_{ij} *sont des matrices rectangulaires dont le nombre de colonnes est égal à* m_i *et le nombre de lignes à* m_j.

Théorème 4. *Pour que la matrice stochastique* Φ *soit primitive, il est nécessaire et suffisant que l'ensemble des états possibles contienne une classe essentielle seulement, formée d'une seule sous-classe, indépendamment de la présence ou de l'absence des états non-essentiels.*

Les théorèmes cités suffisent complètement pour établir un lien entre les deux systèmes d'exposer précédemment indiqués. D'ailleurs, la méthode d'exposer admise présente aussi un certain intérêt pour la théorie des matrices non-négatives. Ayant en vue d'étudier d'une manière plus détaillée dans certains articles ultérieurs quelques problèmes et théorèmes de la théorie des chaînes de Markoff, en particulier à une infinité dénombrable d'états possibles en partant du lieu indiqué, nous finirons cette Note en énonçant les théorèmes suivants, qui donnent une illustration de la synthèse proposée.

Théorème 5. *Pour qu'une chaîne de Markoff soit régulière, il faut et il suffit que l'ensemble des états possibles contienne une classe essentielle seulement et formée d'une seule sous-classe.*

Remarque. Pour la régularité positive dans le théorème 5, il faut encore exiger que l'ensemble S ne contienne pas des états non-essentiels.

Théorème 6. *Pour chaque* E_i *de la classe essentielle*

$$\lim_{m\to\infty} \prod_{ii}^{(m)} = \frac{1}{M_{ii}} = \frac{\Phi_{ii}(1)}{\sum \Phi_{jj}} \; *$$

* On peut démontrer que dans le cas considéré tous les $M_{jj}(j = \overline{1,n})$ sont finis.

la sommation suivant l'indice j est effectuée dans uns classe essentielle contenant l'état E_i; la matrice Φ est formée des probabilités transitoires correspondant aux états de la même classe; enfin $\Phi_{ii}^{(1)}$ est le mineur du déterminant $|\lambda E - \Phi|$, où E est la matrice unitaire correspondant à l'élément $\lambda - p_{ii}$ pour $\lambda = 1$.

Théorème 7. *Pour que le principe ergodique soit réalisé, il faut et il suffit que l'ensemble de tous les états possibles contienne une seule classe essentielle formée d'une seule sous-classe indépendamment de la présence ou de l'absence des états non-essentiels.*

Désignons par $S^{(1)}$, $S^{(2)}$, ..., $S^{(r)}$ les classes essentielles de l'ensemble S, et par $S_1^{(a)}$, $S_2^{(a)}$, ..., $S_{d(a)}^{(a)}$ les sous-classes de la classe $S^{(a)}$ $(a = 1, r)$ *.

Nous aurons alors

Théorème 8. *Dans la classe $S^{(a)}$ pour tous les E_i de la sous-classe $S_\beta^{(a)}$ et les E_j de la classe $S_\gamma^{(a)}$ la probabilité $p_{ij}^{(n)}$ tend vers une limite*

$$p_j = \frac{\Phi_{jj}^{(1)}\, d(a)}{\sum \Phi_{jj}^{(1)}}$$

qui ne dépend pas de i quand n tend vers l'infini en parcourant les valeurs

$$n \equiv \gamma - \beta \pmod{d(a)}$$

La sommation suivant l'indice j est effectuée dans la classe $S^{(a)}$, et la matrice Φ est formée des probabilités transitives correspondant aux états de cette classe.

De ce théorème on obtient comme conséquence le théorème connu:

Théorème 9 (1). *La somme des limites p_j pour tous les états de chaque sous-classe est égale à 1.*

Université d'Asie Centrale
à Tachkent

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le 7. VIII. 1944.

LITTÉRATURE CITÉE

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* Indiquons d'ailleurs que l'on peut indiquer un simple algorithme pour déterminer le nombre de sous-classes $d(a)$ de la classe $S^{(a)}$ en utilisant la matrice des probabilités transitoires correspondant aux états de la classe $S^{(a)}$.

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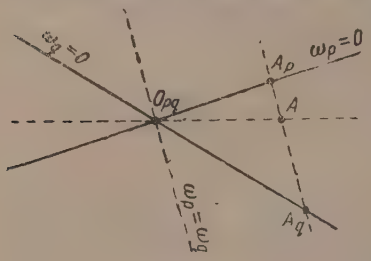
MECHANICS

DIAGRAM OF ANGULAR VELOCITY RATIOS OF A REGULAR TOOTHED GEAR MECHANISM WITH THREE DEGREES OF FREEDOM

By M. KREINES

(Communicated by L. S. Leibenson, Member of the Academy, 28.I.1945)

In a previous paper (1) in which the definition of a regular toothed gear mechanism was given, a diagram of angular velocities was constructed, and its application in the investigation and synthesis of regular mechanisms analysed for the case of mechanisms with two degrees of freedom. In the present paper we give the diagram of angular velocity ratios for regular toothed gear mechanisms with three degrees of freedom, and an analysis of its application in the investigation and synthesis of such mechanisms.



Consider a regular toothed gear mechanism with three degrees of freedom. Let its main links be 1, 2, ..., n; then the relations set up by such a mechanism between the angular velocities $\omega_1, \omega_2, \ldots, \omega_n$ of its main links 1, 2, 3, ..., n, may be expressed by the following system of equations:

$$\begin{aligned} \omega_1 &= a_1 u + b_1 v + c_1 w \\ \omega_2 &= a_2 u + b_2 v + c_2 w \\ &\cdots\cdots\cdots\cdots \\ \omega_n &= a_n u + b_n v + c_n w \end{aligned} \qquad (1)$$

where u, v and w are three independent parameters, and $a_1, a_2, \ldots, b_1, b_2, \ldots, c_1, c_2, \ldots$ are constant coefficients depending on the design of the mechanism and the choice of the parameters u, v and w.

Let u, v, w be the homogeneous coordinates of a point in a projective plane. Now, using the Cartesian system of coordinates xOy, let us represent the three numbers u, v and w as a point with the coordinates $x = u/w$ and $y = v/w$.

Now draw the lines $\omega_1 = 0$; $\omega_2 = 0$, ..., $\omega_n = 0$ on the diagram and let those points on the plane at which three or more of these lines intersect be called junction points. Draw in another series of lines: $\omega_1 = \omega_2$, $\omega_1 = \omega_3$, ..., $\omega_{n-1} = \omega_n$; now the resulting drawing may be called the diagram of angular velocity ratios for the regular toothed gear mechanism with three degrees of freedom under consideration.

This angular velocity ratio diagram has the following properties:

1. Each point of the diagram corresponds to one definite value of the angular velocity ratio of any two main links of the mechanism. In order to determine the value of the ratio ω_p/ω_q corresponding to a given point A (see figure), consider the links: $\omega_p = 0$ and $\omega_q = 0$; let their intersection point be O_{pq}; then the value of the ratio $\left.\frac{\omega_p}{\omega_q}\right|_A$ that is to be found equals the complex ratio of the straight lines $\omega_p = 0$, $\omega_q = 0$, $O_{pq}A$ and $\omega_p = \omega_q$. For instance, the value of $\left.\frac{\omega_p}{\omega_q}\right|_A$ can be determined by drawing a straight line through A, parallel to the line $\omega_p = \omega_q$; then $\left.\frac{\omega_p}{\omega_q}\right|_A$ is equal to the ratio between the directed intercepts AA_p and AA_q of this line.

2. Any pair of straight lines $\omega_p = 0$ and $\omega_q = 0$ $(p \neq q)$ coincide only if a constant, unchanging transmission ratio has been established between the links p and q.

3. Any three straight lines $\omega_p = 0$, $\omega_q = 0$ and $\omega_r = 0$ intersect at the same junction point only if a relation of the type $a\omega_p + b\omega_q + c\omega_r = 0$ (where a, b and c are constants depending upon the design of the mechanism) has been established between the angular velocities of the links p, q and r. When this is so, the ratio of the coefficients b and a, for instance, equals the complex ratio of the straight lines $\omega_p = 0$, $\omega_q = 0$, $\omega_r = 0$ and $\omega_p = \omega_q$ with the sign reversed.

4. The coefficients a, b and c satisfy the equation $a + b + c = 0$, and, therefore, the equation $a\omega_p + b\omega_q + c\omega_r = 0$ may be written as $\omega_p - \omega_r = i(\omega_q - \omega_r)$, where i is a rational number, depending upon the design of the mechanism, only in the case where all three lines $\omega_p = \omega_q$, $\omega_q = \omega_r$ and $\omega_r = \omega_p$ coincide *).

5. The mechanism under consideration has no constant external moment fulcrum in the single case where all the lines $\omega_1 = \omega_2$, $\omega_1 = \omega_3$, ..., $\omega_{n-1} = \omega_n$ intersect at one point.

6. Having all the lines $\omega_1 = 0$, $\omega_2 = 0$, ..., $\omega_n = 0$ and $\omega_1 = \omega_2$, ..., $\omega_1 = \omega_n$, any of the lines $\omega_p = \omega_q$ (where $p, q = 1, 2, \ldots, n$, and $p \neq q$) can be plotted.

7. The angular velocity ratio diagram of a regular mechanism with three degrees of freedom can be used for plotting the angular velocity diagram of any mechanism with two degrees of freedom, formed from the one under consideration by stopping one of its main links. In order to draw the angular velocity diagram of the mechanism formed by stopping any link p, it is necessary to choose at random the origin and positive direction on the line $\omega_p = 0$, then this line is taken as the OU axis, and a line drawn perpendicular to it — as the $O\Omega$ axis.

Draw any straight line parallel to $\omega_p = 0$; let its equation be $\omega'_p \equiv a'_p u + b'_p v + c'_p w = 0$. Now consider one of the lines $\omega_q = 0$ $(q = 1, 2, \ldots, n;\ q \neq p)$ and let it intersect $\omega_p = 0$ at A. Draw a perpendicular from the point of intersection of $\omega'_p = \omega_q$ and $\omega_p = 0$ to $\Omega = 0$; find a point B_q on this perpendicular, whose ordinate Ω equals 1.

Draw a straight line through A_q and B_q. Repeat this for all values of $q \neq p$. The lines thus plotted in the system of coordinates $UO\Omega$ are the angular velocity graphs for the main links of a mechanism with two degrees of freedom, formed from the one under consideration by stopping one of its links p. All these graphs are drawn under the assumption that ω'_p is taken as unity for measuring the angular velocities.

If the mechanism with two degrees of freedom is formed from the one under consideration by throwing in a clutch to connect two of its links p_1 and p_2, then all the constructions described above should be done on the basis of the line $\omega_{p_1} = \omega_{p_2}$, which in this case takes the part of the line $\omega_p = 0$ used when the link p was stopped.

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* It will be noted that coincidence of all three lines necessarily follows from the coincidence of any two of them.

8. Suppose all the lines $\omega_{p_1}=0$, $\omega_{p_2}=0$, ..., $\omega_{p_k}=0$ have a common point of intersection; then the angular velocity ratio diagram makes it possible to plot the basis (1) of the angular velocity diagram of anyone of those mechanisms with two degrees of freedom which respond to the relationship, the mechanism under consideration sets up between the angular velocities $\omega_{p_1}, \omega_{p_2}, \ldots, \omega_{p_k}$. In order to plot this basis for the angular velocity diagram, taking, for instance, link p_1 as the planet carrier, all the lines must be removed, except $\omega_{p_1}=0$, $\omega_{p_2}=0$, ..., $\omega_{p_k}=0$ and $\omega_{p_1}=\omega_{p_2}, \ldots, \omega_{p_1}=\omega_{p_k}$, and then make the same constructions as in 7 (see above), beginning with any line taken at random parallel to $\omega_{p_1}=0$; in this case ω_{p_1} is taken as unity for measuring the angular velocities.

In applying the above to the synthesis of regular tooth gear mechanisms with three degrees of freedom, write out the system of equations (1) in accordance with the purpose for which the mechanism is designed, and plot the lines $\omega_p=0$ $(p=1, 2, \ldots, n)$ and $\omega_p=\omega_q$ $(p, q=1, 2, 3, \ldots, n;\ p\neq q)$. Then examine the system S of equations formed (see 3) at the various junction points in the diagram; if this system consists of $n-3$ independent equations, the diagram thus plotted is the angular velocity ratio diagram of the mechanism to be made; but if it contains less than $n-3$ independent equations, then new junction points must be found by selecting auxiliary lines $\omega_{n+1}=0$, $\omega_{n+2}=0$, ..., $\omega_{n+m}=0$ and adding them to the diagram, so as to make the total number of independent equations, formed at the various junction points, equal to $n+m-3$.

The resulting drawing is the angular velocity ratio diagram of the mechanism to be made. From this angular velocity ratio diagram the angular velocity diagrams of the mechanisms mentioned in 8 should be plotted; then these mechanisms themselves are designed, after which their like links are joined to form the mechanism in question.

In concluding we may note that quite obviously the above train of reasoning can comparatively simply be extended to mechanisms with more than three degrees of freedom. However, the necessity of having to do with a space diagram in this case introduces a serious complication.

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AERODYNAMICS

CONDENSATIONAL JUMPS

By S. Z. BELENKY

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The densification jumps known to form after the narrow section of Laval nozzles result in disturbances in the flow coming from the nozzle, which create serious difficulties for aerodynamic investigations.

Until recently it was thought that the chief cause of these jumps was disintegration of the supersonic flow as a result of the insufficiently stream-line shape of the walls of the nozzle in the region of the narrow section; thus, the nature of these jumps was supposed to be identical with Riemann's shock waves, which were studied in detail by Rankin and Hugoniot(1).

However, the experimental works of Wieselsberger and of his collaborators (chiefly, those of Hermann) showed that these densification jumps in Laval nozzles depend on the humidity of the air sucked into the nozzle. Particularly, it was established, that if the air was dried before being fed to the nozzle, these jumps disappeared.

Hence the authors struck upon the idea of connecting the jumps in the wide part of Laval nozzles with the sudden condensation of the water vapour in the air. This sudden condensation of water vapour is of fundamental importance not only for air-flows in Laval nozzles, but generally for the aerodynamics of high speeds as well. This phenomenon should be taken into account in the case of great supersonic velocity of air-flow along the bodies.

It is not our intention in this paper to go into the conditions under which condensation may set in; this will be dealt with in another report*. We shall only assume, in accordance with experimental observation, that a sharp boundary arises in the nozzle, separating the air which contains uncodensed water vapour from that in which the vapour has condensed, this boundary being observed as a «jump».

Let the density, velocity, pressure and heat function of the air before the condensational jump be, respectively: ρ_1, v_1, p_1 and i_1, and after the jump—ρ_2, v_2 p_2 and i_2. The gas flow is adiabatic before and after the jumps. The change in mass, of the gas as a result of condensation may be neglected, since water vapour makes up only a small fraction of the mass of the gas as a whole. The chief effect which should be taken into account is the liberation of the heat of vaporization, and to some extent, the heat of fusion, since patricles of ice may even form after the narrow section of the Laval nozzle at low temperatures. The liberation of this latent heat results in a change in the balance of energy. We shall deal with a unidimensional current, *i. e.* with a «direct condensational jump», according to the conceptions developed by Hermann (2).

* The problem of the conditions under which condensation sets in in Laval nozzles is dealt with in a paper by Oswatitsch [3]. This paper will also be discussed somewhat later.

The fundamental equations of gas dynamics take the following form for our case:

a) the equation of continuity

$$\rho_1 v_1 = \rho_2 v_2 \tag{1}$$

b) the equation of the conservation of motion:

$$\rho_1 v_1 (v_1 - v_2) = p_2 - p_1 \tag{2}$$

c) the equation of the conservation of energy

$$i_1 + {}^1/_2 v_1^2 = i_2 + {}^1/_2 v^2 - \delta i_0 \tag{3}$$

$$\delta i_0 = \alpha r \tag{4}$$

where α is the ratio between the quantity of water vapour condensed during the jump and the total quantity of gas (air and water vapour); r is the latent heat liberated during condensation.

Applying the equation of state in the form of $i = \frac{k}{k-1}\frac{p}{\rho}$, where $k = \frac{c_p}{c_v}$, p and ρ can be excluded from equations (1), (2) and (3), and we get the following expression which connects up the velocity before and after the jump:

$$v_2 v_1 - \rho_2 v_1^2 - v_2 a_1^{*2} + v_1 a_2^{*2} = 0 \tag{5}$$

where a_1^* and a_2^* are the «critical velocities» before and after the jump. The critical velocity is in a simple relation to the «total heat function» i_0, where $i_0 = i + {}^1/_2 v^2$

$$a^{*2} = 2\frac{k-1}{k+1} i_0 \tag{6}$$

In the ordinary Riemann jump the critical velocity does not change during the jump, since i_0 remains constant. In our case, however, the change in i_0 involves a change in the critical velocity. For purposes of further investigation it is convenient to measure velocity in units of $\lambda_1 = v_1/a_1^*$ and $\lambda_2 = v_2/a^*$. In such units the field of presonic velocities corresponds to $\lambda < 1$, and the field of supersonic velocities, to $\lambda > 1$. Then equation (5) may be written as follows:

$$\lambda_2^2 - \lambda_2 \gamma \frac{\lambda_1^2 + 1}{\lambda_1} + 1 = 0, \quad \gamma = a_1^*/a_2^*. \tag{7}$$

During the jump the total heat function undergoes a change equal to δi_0. Let $\beta = \delta i_0/i_0$. It can easily be seen [see equation (6)] that γ can very simply be expressed in terms of β

$$\gamma^2 = \frac{1}{1+\beta} \tag{8}$$

It must be noted that in this theory α [see equation (4)] and, consequently, β and γ also, remain indefinite parameters, whose magnitude can be determined either from a more precise theory which should deal with the structure of the condensational jump, or from experiment.

Solving equation (7) for λ_2 we get:

$$\lambda_2 = \frac{1}{2}\frac{\gamma(\lambda_1^2+1)}{\lambda_1} \pm \sqrt{\frac{\gamma^2(\lambda_1^2+1)^2}{4\lambda_1^2} - 1} \tag{9}$$

Hence it may immediately be seen that for a given value of $\lambda_1 \gamma$ cannot be less than

$$\gamma_{min} = 2\lambda_1/(1+\lambda_1^2) \tag{10}$$

and β cannot be greater than

$$\beta_{max} = \frac{\delta i_0}{i_0} = \left(\frac{\lambda_1^2 - 1}{2\lambda_1}\right)^2 \tag{11}$$

In the vicinity of the narrow section of Laval's nozzle this value is not very large. For instance, if $\lambda_1 \cong 1.2$ or Mach's number equals 1.26 *, then $\beta_{max} = 0.0336$.

Now, with the aid of equation (9) let us consider the possible types of condensational jumps.

1) Suppose $\lambda_1 > 1$ and that the sign before the root is positive. After the jump the velocity λ_2 will vary from $\lambda_2 = \lambda_1$ when $\gamma = 1$ to $\lambda_2 = 1$ when $\gamma = \gamma_{min}$. Thus, after the jump the velocity remains supersonic all the time (a feature which distinguishes this jump essentially from ordinary condensational jumps), decreasing in inverse proportion to α. The increase in pressure during the jump, which characterizes the intensity of the latter, can be determined from formula (2). The intensity reaches its maximum when $\gamma = \gamma_{min}$ (or $\beta = \beta_{max}$) and is equal, as can easily be seen, to half the intensity of a direct Riemann densification jump, which takes place when the velocity is equal to λ_1. The minimum intensity of the jump is zero, which is the case when no condensation takes place ($\alpha = \beta = 0$, $\gamma = 1$). Evidently, jumps of precisely this kind are observed usually after the narrow section of Laval nozzles.

2) Suppose $\lambda_1 > 1$ and that the sign before the radicle is negative. After the condensational jump velocity λ_2 will vary from $\lambda_2 = 1/\lambda_1$ when $\gamma = 1$ to $\lambda_2 = 1$ when $\gamma = \gamma_{min}$, but will remain presonic all the time.

The velocity λ_2 increases in direct proportion with α. The intensity reaches its maximum when $\gamma = 1$ and equals the intensity of a Riemann densification jump for a velocity of λ_1, while the minimum intensity, attained when $\beta = \beta_{max}$, equals one half of the maximum intensity. The case under consideration probably corresponds to a coincidence of two jumps at one point, *viz.* a Riemann densification jump and a condensational jump. Since these jumps are due to different physical causes, a coincidence between them is quite improbable, though possible from the standpoint of the general conservation laws (1), (2), (3). Nevertheless these jumps may occur under definite conditions.

That the densification jumps observed in Hermann's [2] experiments cannot be of the same type as those considered here, becomes evident already from the fact that they disappear when the nozzle is dried, while the jumps we have in view, attain a maximum intensity precisely in the absence of condensation.

3. Suppose $\lambda_1 < 1$ and that the sign before the radicle is positive. The velocity λ_2 will vary from $\lambda_2 = \lambda_1$ when $\gamma = 1$ to $\lambda_2 = 1$ when $\gamma = \gamma_{min}$, always remaining presonic. Before the jump the velocity λ_2 is all the time greater than the velocity λ_1.

4. Suppose that $\lambda_1 < 1$ and that the radicle is taken with the plus sign. The velocity λ_1 will vary from $\lambda_2 = 1/\lambda_1$ when $\gamma = 1$ to λ_2 when $\gamma = \gamma_{min}$, always remaining supersonic. In this case we probably have again a coincidence between Riemann's rarefaction jump and condensational jump. It is known that Riemann's jumps (rarefaction jumps) cannot take place in the presonic region, because they bring about a decrease in entropy. From this point of view some rarefaction jumps of the third and fourth type call forth special consideration, however. So far rarefaction jumps have never been observed, probably because their appearance in the nozzle requires that the incoming air have a high moisture content.

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* The relation between λ and Mach's number can be seen from the following equation: $\lambda^2 = \frac{k+1}{2} \frac{Ma^2}{1 + \frac{k-1}{2} Ma^2}$. When Ma tends to infinity, $\lambda^2 = \frac{k+1}{k-1}$.

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PHYSICS

GEOMETRICAL SELECTION IN A GROWING CRYSTAL AGGREGATE

By G. G. LAEMMLEIN

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Simultaneous growth of a large number of crystals, oriented irregularly over any surface, finally results in the formation of an aggregate showing some kind of texture fibrous, columnar, or radial. This phenomenon is well known in mineralogy and metallurgy, where the columnar crystalline crust which grows inward from the walls of the form towards the heart of the casing, has been termed the «zone of transcrystallization».

The formation of such an ordered texture of growth is due to anisotropy of the velocities of growth in the crystal grains. Only those of the irregularly oriented crystals formed at some surface remain intact in the textural aggregate whose directions of maximum growth rates either coincide with or but slightly deviate from the normal to the initial crystallization surface (the basis surface). The growth of the rest of the grains will gradually be inhibited by these more fortunalety oriented crystals, and the greater the angle between the vector of the greatest rate of growth of the crystal and the normal to the basis surface, the sooner this will take place.

Many methods for the artificial production of single crystals are based upon this purely «geometrical principle of selection» ([1–3]). On the other hand, it seems to me that a clear conception of the principles underlying geometrical selection in crystals ought to be of considerable help in investigations concerning the elimination of large-grained transcrystallization zones in metal castings.

The formation of growth textures has often been ascribed to various external conditions of growth, particularly to the orienting influence of the direction of heat conduction. However, if we regard the growth of a crystal under any conditions as being, above all, a process by virtue of which a definite geometrical space is filled, we find that this basic, purely geometrical element of growth determines fully all the phenomena connected with the formation of growth texture. External conditions can influence this phenomenon only indirectly, whether positively of negatively, varying the rate of growth of the face and the shape of the growing crystal.

It seemed a matter of interest to make a more elaborate analysis of the way in which the number of growing grains decreases and in which the texture becomes more regular, as the process of crystal growth gets farther away from its starting point.

Such observations and calculations were made for growth texture preparations obtained by crystallizing out a supercooled fusion of thymol ($C_{10}H_{14}O$, m. p. 44°C) in a thin wedge-shaped layer between two glasses. Crystallization was excided by briskly drawing a needle infected with pulverized thymol along the thin

edge of the preparation. Such treatment gives rise to slightly dendritic rhombohedric crystals, usually flattened out along one of the rhombohedron faces (1011).

In three experiments a preparation 45 mm in length gave respectively 105, 127 and 140 initial crystallization centres. The length of each grain was measured under the microscope in polarized light. The data obtained in this way were used to plot a diagram (Fig. 1), in which the distance from the initial point of growth (in units of the average distance on the basis line between one nucleus and the next) was plotted along the abscissa axis, and the percentage of crystals which

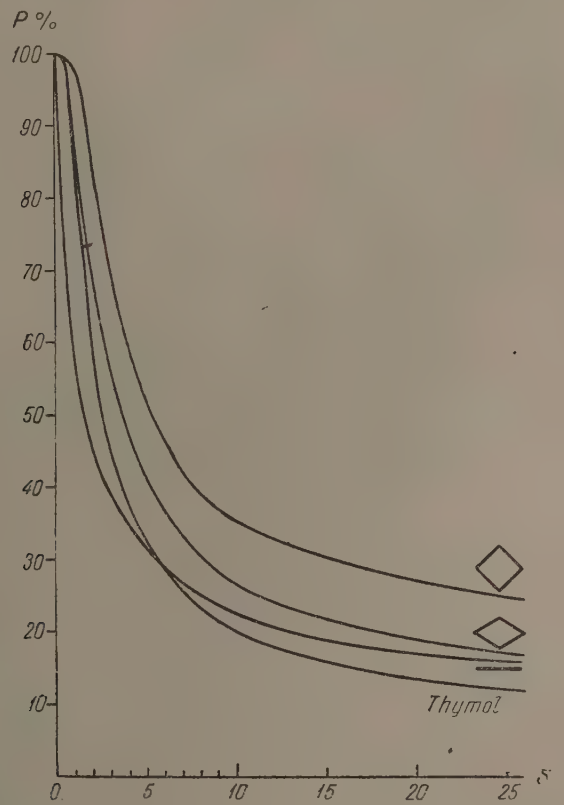

Fig. 1. The decrease of the number of growing «crystals». The per cent P of «surviving» crystals is plotted along the ordinate axis, and the distance S from the starting point of growth (in units of the distance between the neighbouring centres), along the abscissa axis

had grown out to that distance, along the ordinate axis. In all theree cases the curves were characteristic, beginning with 100 per cent at the ordinate zero, and then dipping sharply downwards at a distance of one unit, this corresponding to the stage when the grains are growing up to one another to form a continuous front of growth, and begin geometrical competition. At a distance of only three units the curve has already fallen to nearly 50 per cent; after this it becomes less steep, and at a distance of 10 units it reaches 20 per cent, and then, declining more and more gently, it gradually approaches zero or some minimum, depending upon the percentage of crystals which have their maximum growth velocities directed along the normal to the edge of the preparation (the basis line). Evidently, as the limit is approached, only one growing crystal should remain within the growing space.

This rapid decrease in the number of grains taking part in the growth explains the origin of the thin fine-grained layer, described in metallurgy to surround the columnar transcrystallization zone on its outside and, apparently, to form an integral part of the latter.

In order to bring to light the statistic nature of the process of geometrical selection, the author turned from the experimental flat model of growth texture formation to an examination of a strictly two-dimensional graphical representation of such a process. A two-dimensional representation of this kind, closely reproducing the experiment of crystallization in a wedge-shaped layer as described above, can, of course, serve only as a first approximation, but quite a definite one, nevertheless, of the volumetric picture of the struggle between the growing crystals.

In order to simplify the graphical constructions, it was assumed that the initial growth points were distributed evenly along a straight basis line. Hence, the distances between these points should correspond to the average distance between the crystal nuclei in the experiment.

The random orientation of the directions of maximum growth ratio in respect to the basis line was selected from a table of random numbers with an error not exceeding 1 per cent. All the «crystals» were assumed to begin their growth simultaneously, and they were all assumed to be of identical simple shape and to have equal rates of growth in like directions.

The growth boundaries of each crystal were found on a large-scale drawing. As we know, the sines of the angles between the contacting faces of two adjacent crystals and the geometrical boundary between them are to each other as the rates of growth of these faces. If the rates of growth of the contacting faces are equal, the boundary between the crystals is the bisector of the angle between these faces. Two such growth pictures were drawn out to a distance of 25 units, each of them involving one hundred crystals*.

One of the drawings represented the growth of a hundred «crystals» in the form of rhombs with acute angles of 60°. The curve of decrease of the number of «survivors» in this «struggle for space» is given in Fig. 1, and is quite analogous to the curve described above, obtained in the experiments with thymol. The only difference is that in the initial stages the number of thymol grains decreased somewhat more rapidly. This discrepancy is probably due to the dimensions of the grains being commensurable with the thickness of the fusion layer, as a result of which the third dimension of the layer still affects the process.

The other drawing also represents the growth of one hundred crystals with the same orientation, but in the form of squares. This change in form (increase of the vertex angle and doubling of the number of competitive directions) resulted in the decrease of «surviving» crystals being somewhat slowed down (Fig. 1).

Evidently, as the angle between the «faces» of the growing aggregate and the number of equal directions increase, the decrease curve becomes less and less steep. This phenomenon manifests itself especially in the early stage of bend in the curve, which begins further and further away from the starting point, *i. e.* depression of the first crystals of the closed growth front as a result of geometrical struggle begins later and later. The limit is reached when all the directions of growth of the crystal are equal, and then geometrical struggle will result in the formation of an aggregate with parallel fibres of a number equal to the number of initial growth centres.

The other limit—when the angle between the faces of the aggregate is at its least—can be studied in a construction in which «needles» are substituted for the «crystals», these «needles» being the vectors of the maximum rate of growth. Evidently, the decrease curve for this case, given also in Fig. 1, is at its minimum values in the strictly two-dimensional image of geometrical selection. In particular, the stage in which the crystals grow up to one another, and the early stage

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* These extremely precise graphical constructions were executed by M. K. Zhitkova, without whose help the work could hardly have been done at all.

of bend in the curve, are absent in this case; the curve begins to fall steeply right from the start.

The graphical construction of the picture of growth of «needles» was so simple, that it was possible to increase their number to 400 and to follow their growth through to a distance of 40 units. Such a large number of growing «crystals» made it possible to study the distribution of frequencies of crystals of various orientation at various distances from the starting point of growth. The tridimensional

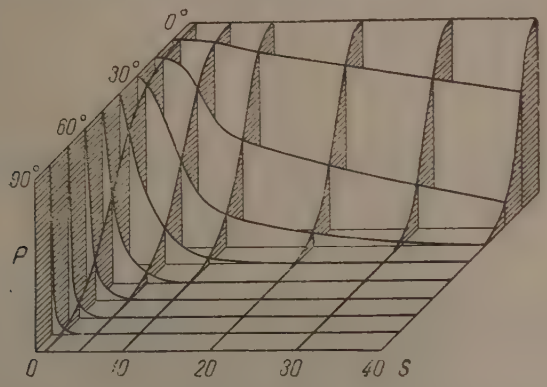

Fig. 2. The relation between the number of growing needle crystals and their angle of slope to the basis line. The angle of deflection from the normal towards the basis line is plotted along the first axis; the distance S from the starting point of growth, along the second; and the per cent P of «surviving» crystals, along the third

diagram in Fig. 2 shows the relation between the distribution of frequencies of occurrence and the angle of slope towards the basis line, and the distance from the starting point of growth. As may have been expected, by the time a distance of 20 units is reached, there are almost no needles left at an angle of more than 40° to the normal. At a distance of 40 units from the starting point of growth, the only needles left are those whose angle of deflection is not more than 30°. After this, the texture will evidently become regular faster and faster.

This limit, although practically impossible, gives, however, quite a descriptive, statistic geometrical conception of the details of the process of geometrical selection which takes place in realty.

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PHYSICS

INITIAL BOUNDARY PROBLEMS OF ELECTRODYNAMICS

By J. N. FELD

(Communicated by N. D. Papalexi, Member of the Academy, 4. IX. 1944)

The object of the present paper is to formulate and solve certain initial boundary problems of electrodynamics for any unsteady oscillations.

By the first I. B. P. E. (initial boundary problem of electrodynamics) is to be understood the problem in which the electromagnetic field in a space v, bounded by a close geometrical surface s, is to be determined for an instant $\tau = t^*$ from the following data:

1) On the surface s is given the tangential component of the electric vector for $0 \leqslant \tau \leqslant t$.

2) Inside v are given the values of the field vectors for $\tau = 0$

$$\vec{E} = \vec{E}_0, \ \vec{H} = \vec{H}_0 \text{ for } \tau = 0 \tag{1}$$

3) If inside v there are sources of the field, they are supposed to be known.

4) If the space v extends to infinity, all sources are supposed to be located at a finite distance.

The second I. B. P. E. is formulated similarly with the only difference that a magnetic rather than electric vector figures in point 1.

Finally, if the tangential component of the electric vector is given on a portion s_1 of the surface s and the tangential component of the magnetic vector is given on the rest of the surface (portion s_2), the problem is said to be a mixed I. B. P. E. By virtue of the uniqueness theorem (1) all these problems have a unique solution.

For solving the I. B. P. E., we will make use of the generalized lemma of Lorentz obtained by the author previously (2); yet we have first extended it to the case where the initial conditions of type (1) are set up. Employing the method used in the earlier work (2) and proceeding upon more general initial conditions** (1), we shall obtain the following expression for the generalized lemma of Lorentz:

$$\int_0^t \{\operatorname{div}[\vec{E}(t-\tau)\vec{H}_1(\tau)] - \operatorname{div}[\vec{E}_1(\tau)\vec{H}(t-\tau)]\}\, d\tau =$$

$$= \frac{\mu}{c}\{\vec{H}_0\vec{H}_1(t) - \vec{H}_{1,0}\vec{H}(t)\} + \frac{\varepsilon}{c}\{\vec{E}_{1,0}\vec{E}(t) - \vec{E}_0 E_1(t)\}. \tag{2}$$

Here $\vec{E}$, $\vec{H}$ and $\vec{E}_1$, $\vec{H}_1$ are two arbitrary fields satisfying the Maxwell equations, and $\vec{E}_0$, $\vec{H}_0$ and $\vec{E}_{1,0}$, $\vec{H}_{1,0}$ are their values for $\tau = 0$. Proceeding to the solution of the first I. B. P. E., we shall suppose the presence within v of

* τ is the current coordinate of time; t is an arbitrary fixed coordinate of time.

** In (2) it was assumed that $\vec{E}_0 = \vec{H}_0 = 0$.

point sources — electric dipoles with moments $\vec{p}_n$, located at points q_n ($n = 1, 2, 3, \ldots, k$).

The conductivity σ inside v is put at 0.

We will now use the Lemma (2), in which $\vec{E}$, $\vec{H}$ are understood to denote the field we are to find, and $\vec{E}_1$, $\vec{H}_1$, the field of auxiliary electric dipole placed at the point q' of observation. The field of the auxiliary electric dipole must satisfy the following conditions.

I. The tangential component of the vector $\vec{E}_1$ vanishes at surface s.

II. $\vec{E}_{1,0} = \vec{H}_{1,0} = 0$ iniside v.

III. The moment $\vec{p}$ of the auxiliary dipole has a constant direction and is equal to

$$p(\tau) = \begin{cases} 0 & \text{for } \tau < 0 \\ \frac{1}{\lambda}\left(\tau - \frac{1}{2\pi}\sin\frac{2\pi}{\lambda}\tau\right) & \text{for } 0 \leqslant \tau \leqslant \lambda \\ 1 & \text{for } \tau > \lambda \end{cases} \tag{3}$$

Here λ is an arbitrary positive parameter. The function $p(\tau)$ is continuous up to the second derivative inclusive. Taking all of this into account, we will integrate the equality (2) over v, having preliminarily excluded from the volume v the points where the sources q_n and the auxiliary dipole q' are located. The exclusion of these points is effected by means of infinitely small spheres. After lengthy, though not very complicated, transformations we shall find

$$\left\{\int_0^t \vec{p}'(\tau)\,\vec{E}(t-\tau)\,d\tau - \frac{1}{3}\vec{p}(t)\vec{E}_0\right\}_{q'} -$$

$$-\sum_{n=1}^{k}\left\{\int_0^t \vec{p}_n'(\tau)\,\vec{E}_1(t-\tau)\,d\tau + \frac{1}{3}\vec{p}_n(0)\,\vec{E}_1(t)\right\}_{q_n} =$$

$$= \frac{1}{4\pi}\int_{(v)}\left\{\mu\vec{H}_0\vec{H}(t) - \varepsilon\vec{E}_0\vec{E}_1(t)\right\}dv - \frac{c}{4\pi}\int_0^t d\tau\int_{(s)}[\vec{E}(t-\tau)\,\vec{H}_1(\tau)]\vec{ds} \tag{4}$$

The volume integral which enters in this and the subsequent formulae (6) and (7) is to be understood in the sense of the «principal value» of Cauchy. The subscripts q' and q_n on the left-hand side of (4) indicate that the values of the quantities within the brackets are taken at points q' and q_n, respectively. The vectors $\vec{p}$, $\vec{p}'(\tau)$, $\vec{E}_1$, $\vec{H}_1$ depend explicitly or implicitly upon the arbitrary positive parameter λ (see (3)). Therefore making use of the equality

$$\left.\begin{aligned} E_p(t)\lim_{\lambda\to 0}\int_0^t \vec{p}'(\tau)\vec{E}(t-\tau)\,d\tau \\ \lim p(t) = 1 \text{ for } t > 0 \end{aligned}\right\} \tag{5}$$

which can be easily proved by means of (3), and proceeding in (4) to the limit for $\lambda \to 0$, we find

$$E_p(t, q') = \frac{1}{3}E_p(0, q') - \lim_{\lambda\to 0}\frac{c}{4\pi}\int_0^t d\tau\int_{(s)}[\vec{E}(t-\tau)\,\vec{H}_1(\tau)]\vec{ds} +$$

$$+\lim_{\lambda\to 0}\left[\frac{1}{4\pi}\int_{(v)}\{\mu\vec{H}_0\vec{H}_2(t) - \varepsilon\vec{E}_0\vec{E}_1(t)\}\,dv +\right.$$

$$\left.+\sum_{n=1}^{k}\left\{\int_0^t \vec{p}_n'(\tau)\vec{E}_1(t-\tau)\,d\tau + \frac{1}{3}\vec{p}_n(0)\,\vec{E}_1(t)\right\}_{q_n}\right] \tag{6}$$

Here $E_p(t, q')$ is the projection of the vector $\vec{E}$ on to the direction of the moment $\vec{p}$ of the auxiliary dipole. Unlike the rest of our formulae, this one brings out an explicit dependence of E_p not merely upon the time, but also upon the observation point q'.

Replacing the auxiliary electrical dipole by a magnetic one with a moment $\vec{m}$ and by the field vectors $\vec{E}_2$, $\vec{H}_2$ satisfying the same conditions I, II, III as in the case of the electric dipole, we find as before:

$$H_m(t, q') = \frac{1}{3} H_m(0, q') = \lim_{\lambda \to 0} \frac{c}{4\pi} \int_0^t d\tau \int_{(s)} [\vec{E}(t-\tau)\, \vec{H}_2(\tau)]\, \vec{ds} -$$

$$- \lim_{\lambda \to 0} \left[\frac{1}{4\pi} \int_{(v)} \{\mu \vec{H}_0 \vec{H}_2(t) - \varepsilon \vec{E}_0 \vec{E}_2(t)\}\, dv + \right.$$

$$\left. + \sum_{n=1}^{k} \left\{ \int_0^t \vec{p}'_n(\tau)\, \vec{E}_2(t-\tau)\, d\tau + \frac{3}{1} \vec{p}_n(0) \vec{E}_2(t) \right\}_{q_n} \right] \qquad (7)$$

Since the place of the auxiliary dipoles and the direction of their moments are arbitrary, formulae (6) and (7) solve our problem determinig the field by the given boundary and initial conditions. The limits of the function (3) and of its derivatives (4) for $\lambda \to 0$ represent «a unit function» and the impulse function of Dirac. Therefore, by taking the vectors $\vec{E}_1$, $\vec{H}_1$ and $\vec{E}_2$, $\vec{H}_2$ to mean the fields created by those dipoles whose moments vary according to the «unit function» law, the formulae (6) and (7) could have been written without the lim signs.

A similar method can be used for the solution of the second and the mixed I. B. P. E., with the sole difference that in the case of the second problem the vector $\vec{E}_1$ in the boundary condition I could be substituted with the vector $\vec{H}_1$.

In the case of the mixed problem, the condition I is to be formulated as follows. In the portion s_1 of the surface s the tangential component of $\vec{E}_1$ is zero, and in the remaining portion s_2 the tangential component of $\vec{H}_1$ is zero. The difference in the boundary conditions I causes the surface integral in the formulae (6) and (7) to undergo obvious modifications (see Lemma (2)). Otherwise these formulae remain unaltered.

It is well to remark in conclusion that formulae (6) and (7) have been derived on the assumption that the moments of the sources $p_n(\tau)$, as well as their derivatives $p'_p(\tau)$, $p''_n(\tau)$ are continuous. If, however, every jump-like variation is considered as a limit of a continuous variation (which can be done by means of formulae analogous to the formula (3)), it will be possible to use the expressions (6), (7) in a discussion of any instantaneous switching on and off of the sources.

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PHYSICS

STEPPED EXCITATION OF MERCURY BY ELECTRONIC IMPACT

By B. YAVORSKY

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According to Dirac (1), the effective cross-section for the excitation of an atom by electronic impact from a state n to a state p is given by the expression

$$Q_{np}=\frac{8\pi^3 m^2}{h^4}\frac{k}{k'}\sum_{m_s,m_{s'}}|(n,k,m_s|V|p,k',m_{s'})|^2 \qquad (1)$$

Here $\frac{k}{2\pi m}$ and $\frac{k'}{2\pi m}$ are the velocities of the electron before and after the impact; V is the energy of interaction; m_s and $m_{s'}$ are the spins of the incident and the scattered electron, quantised along some axis of a weak field.

The effective cross-section for all angles of scattering is defined as follows:

$$Q=\int_0^{\pi} Q_{np}\sin\delta\, d\delta \qquad (2)$$

where δ is the angle of scattering.

The correct form of wave functions upon a collision between the electron and the atom is obtained, if we take linear combinations of atomic wave functions and plane waves, representing incident and scattered electrons. This gives:

$$\Phi(r_1,r_2,r_3)=e^{ikn_0r_3}\Psi(r_1,r_3)+e^{ikn_0r_1}\Psi(r_2,r_3)+e^{ikn_0r_2}\Psi(r_3,r_1) \qquad (3)$$

Such a form of wave function takes electronic exchange into account. Here Ψ is atomic wave function. Slater (2) showed that the radial part of the wave function of the electron could be given in the form:

$$R(r)=Nr^{n^*-1}e^{-Zr/n^*}\left[1+\frac{A_1}{rZ}+\frac{A_2}{(rZ)^2}+\dots\right], \qquad (4)$$

where $(Z/n^*)^2$ is the ionisation potential in Rh units and

$$A_n=-\binom{n^*}{2n}[(n^*-n)(n^*-n+1)-l(l+1)]A_{n-1} \qquad (5)$$

r is measured in radii of the first Bohr orbite of hydrogen.

Usually all terms of the series beyond the term A_1 are omitted.

Penney ([3]) following Slater calculated (in angular variables) the wave functions for the s electron in an unexcited S state, and for s and p electrons in excited P states:

$$\Psi(s, S) = \frac{R_{00}(r)}{\sqrt{4\pi}} = \frac{1.51}{\sqrt{4\pi}} r^{0.87} e^{-0.88r} \left(1 - \frac{0.92}{r}\right)$$

$$\Psi(s, P) = \frac{R_{01}(r)}{\sqrt{4\pi}} = \frac{3.03}{\sqrt{4\pi}} r^{0.69} e^{-1.18r} \left(1 - \frac{0.50}{r}\right)$$

$$\Psi(p, P) = R_{11}(r) P_1^m(\cos\vartheta)\, e^{im\varphi} =$$

$$= 0.421 \sqrt{\frac{3}{4\pi(1+m)!}}\, r^{0.60} e^{-0.62r} \left(1 + \frac{0.83}{r}\right) P_1^m(\cos\vartheta)\, e^{im\varphi}$$

The wave function for the s electron in excited S state can be calculated in a similar way. The calculation gives:

$$\Psi'(s, S) = \frac{R'_{00}(r)}{\sqrt{4\pi}} = \frac{1.50}{\sqrt{4\pi}} r^{-0.40} e^{-.67r} \left(1 + \frac{0.07}{r}\right)$$

The condition for antisymmetry of full atomic wave functions (including the spin) is fulfilled if one takes combinations of these simple functions.

Houston ([4]) showed that the S level could be defined by the wave function of the form:

$$\Psi_s(r_1, r_2) = R_{00}(r) R_{00}(r_2) \frac{1}{\sqrt{2}} [S_\alpha(1) S_\beta(2) - S_\alpha(2) S_\beta(1)] \qquad (6)$$

where S_α and S_β are the spin functions' corresponding to the orientation of the spin along and across the field, respectively. For the wave functions of four P levels he suggested the form:

$$\Psi_{Pjm} = \sum_{i=1}^{4} a_{imj} \varphi_{im} \qquad (7)$$

where m is the component of j along a weak field removing the degeneration; a_{imj} are coefficients calculated by Penney ([3]), according to the formulae proposed by Houston ([4]), for the different values of m_j and different orientation of the spins. The functions φ_{im} define the cases where the electrons are in all possible s and p states with different orientations of spins, and are given in Houston's ([4]) paper.

Calculating the wave function of an excited S-term of mercury in the form:

$$\Psi'_S(r_1, r_2) = R'_{00}(r_1) R'_{00}(r_2) \frac{1}{\sqrt{2}} [S_\alpha(1) S_\beta(2) - S_\alpha(2) S_\beta(1)] \qquad (8)$$

and substituting in the expression (3) the atomic wave functions (6), (7) and (8), we receive full wave functions of the system in the corresponding states. For the transitions $(6P - 7S)$ the matrix element has the form:

$$(P_j, k, m_s | V | S, k'_1, m_{s'}) =$$

$$= \int \left\{ \sum_{1,2,3} e^{ik\mathbf{n}_0\mathbf{r}_3} \left(\sum_{i=1}^{4} a_{imj} \varphi_{im} \right) S_\alpha(3) \right\} V \left\{ \sum_{1,2,3} e^{ik'\mathbf{n}_1\mathbf{r}_3} \Psi'_S(3) \right\} d\nu$$

Here $\sum_{1,2,3}$ denotes the sum over permutations of electrons 1, 2, 3, as is indicated in expression (3).

Upon summation over spins, this matrix component gives three terms more, which correspond to possible orientation of the spin of the impinging electron before and after the impact:

$$[S_\beta(i);\ S_\beta(i)];\ [S_\alpha(i):\ S_\beta(i)];\ [S_\beta(i);\ S_\alpha(i)] \quad (i = 1, 2, 3)$$

After summation over the permutations of the electrons and over spins we obtain:

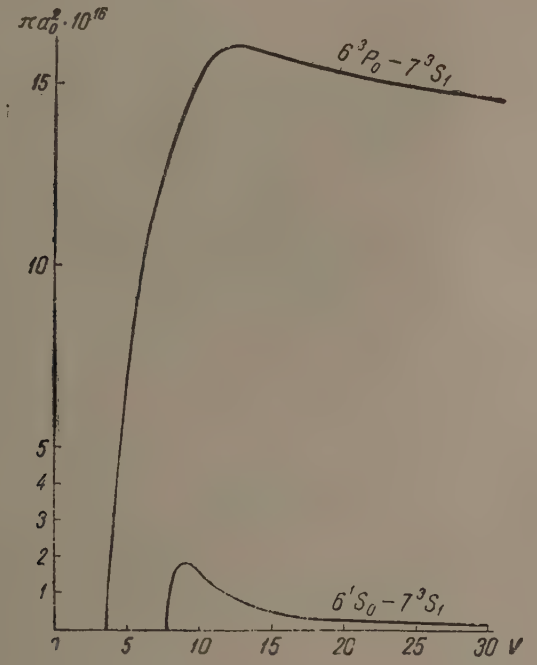
$$Q_{P_j S_m} = \sum_{k}^{k'} \left\{ |a_1 (F_m - G_m)^2 + a_3 g_m|^2 + |a_2 g_{m-1}|^2 + |a_4 g_{m+1}|^2 \right\} \qquad (9)$$

where

$$\left.\begin{aligned} F_m &= 2\pi m \sqrt{2\pi}\, e^2 a_0^2 h^{-2} \int \Psi_{S'}(r_1, r_2)\, V \varphi_{1m}(r_1, r_2)\, e^{i(k n_0 - k' n_1) r_3}\, d\varrho \\ G_m &= 2\pi m \sqrt{2\pi}\, e^2 a_0^2 h^{-2} \int \Psi_{S'}(r_2, r_3)\, V \varphi_{1m}(r_1, r_2)\, e^{i(k n_0 r_3 - k' n_1 r_1)} d\varrho \\ g_m &= 2\pi m \sqrt{2\pi}\, e^2 a_0^2 h^{-2} \int \Psi_{S'}(r_2, r_3)\, V \varphi_{3m}(r_1, r_2)\, e^{i(k n_0 r_3 - k' n_1 r_1)}\, d\varrho \end{aligned}\right\} \qquad (10)$$

$$V = \frac{2}{r_3} - \frac{1}{r_{13}} - \frac{1}{r_{23}}$$

g_{m-1} and g_{m+1} are written down like $g_{m'}$ but contain φ_{2m} and φ_{4m}. The integration over angles has been carried out by the method suggested by Massey and Mohr (5),, similar to the case of stepped excitation of helium (6)



Computed cross-section for the excitation of the 7^3S_1 level of mercury

Upon integration over r integrals of the same type as in the case of helium are met

$$\int_0^\infty \int_0^\infty r_1^2 e^{-\beta r_1} \left(1 + \frac{\gamma}{r_1}\right) J_{3/2}(kr_1)\, \chi_\nu(r_1, r_2)\, r_2^2 e^{-\omega r_2} \left(1 + \frac{\lambda}{r_2}\right) J_{1/2}(k' r_2)\, dr_1 dr_2$$

where

$$\chi_\nu(r_1, r_2) = \begin{cases} \dfrac{r_1^\nu}{r_2^{\nu+1}} & \text{for } r_1 < r_2 \\ \dfrac{r_2^\nu}{r_1^{\nu+1}} & \text{for } r_1 > r_2 \end{cases}$$

If the velocities of the impacting electrons are small, it is possible in calculating to use the asymptotic representation of Bessel functions.

Finally, the cross-section takes the form:

$$Q_{P_jS} = \sum_{m=-l}^{l} \int Q_{P_jS_m} \sin\delta \, d\delta. \qquad (11)$$

By retaining in the series which express the approximate solution of the problem those term which decrease by two orders, it is possible to obtain the effective cross-section for certain levels.

Reliable experimental data on the effective cross-section for straight excitation of the level 7^3S_1 are available in a paper by Hanle and Schaffernicht (7). They give the value of the effective cross-section at the maximum as 1.2 cm^2/cm^3 (1 mm Hg, 0°C). Obtained from the above calculations, the value of the effective diameter of the maximum is 1.80 in these units.

The sharp maximum in the curve, which corresponds to a transition from the single to triplet level, is also in agreement with the experiment. No direct measurements of the effective cross-section have been reported in the literature. The estimation of the great part played by stepped processes in gas discharge and indirect determination of the probability of stepped excitation was made by Fabrikant (8) while studying the intensity of spectral lines in a discharge. The value obtained by him for the effective cross-section at the maximum 12.5 cm^2/cm^3 (1 mm Hg, 0°C) indicates that the effective cross-section for stepped processes is one order higher than for straight excitation. The above calculations confirm this; indeed, we obtained a value of 12.16 for the effective cross-section at the maximum. Upon the excitation of the triplet from the initial triplet state, the curve of the function of excitation changes its form and becomes similar to that of the singlet case. This fact is likewise in agreement with the experiment.

In conclusion I wish to express my gratitude to Professor Fabrikant for having acquainted me with the results of his investigations and for help in carrying out this work.

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TECHNICAL PHYSICS

SPECTRUM ANALYSIS OF METALS BY THE POWDER METHOD AND THE DETERMINATION OF SMALL CONCENTRATIONS OF BORON IN STEELS

By O. B. FALKOVA

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Many of the alloys used in modern metallurgy contain exceedingly small admixtures of various elements, amounting to hundredths and even thousandth of one per cent. Thus, for instance, some modern steels are alloyed with thousandths of a per cent of boron, zirconium, *etc.*

Such infinitesimal quantities of admixtures are very difficult to analyse if the usual chemical methods are employed, and therefore it is quite natural to find spectral methods in common use for these purposes. However, the spectral method has also met with a number of obstacles in this case. First of all, the method is often not sensitive enough. For instance, boron can be derermined in steels by the arc method down to concentrations no lower than 0.01 per cent, and by the spark method—no lower than 0.1 per cent. The sensitivity cannot, as a rule, be increased by increasing the exposure time, since the intensity of the lines is but slightly higher than that of the continuous spectrum.

The next difficulty is met in the preparation of standards for the quantitative analyses. In most cases chemical methods of analysis are incapable of giving the concentration in the standard to a sufficient degree of accuracy. As a rule, moreover, different chemical methods give different absolute values for the content of the admixture. This difficulty is of so serious nature, that it has been repeatedly suggested in literature that results of spectroscopic measurements should be referred directly to the physical-mechanical properties of the metal (electrical conductivity, hardness, *etc.*), rather than to the concentration of the element in the sample expressed in per cent against standards analysed by chemical means.

Finally, with very low concentrations the admixtures are often irregularly distributed throughout the sample: in such cases spectral methods, in which only very small portions of the sample's surface are analysed, are liable to give large errors.

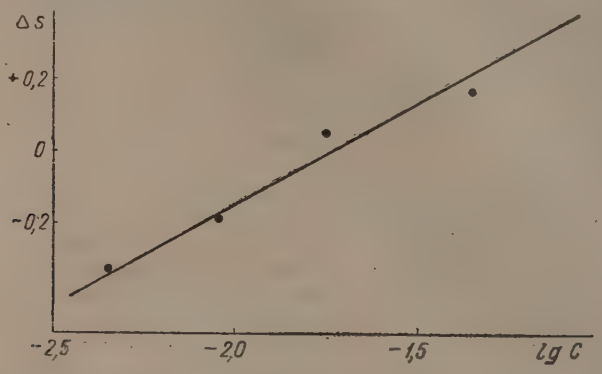
To our opinion, all these obstacles can be avoided in the majority of cases, if the sample to be analysed is preliminarily reduced to powder. We have developed this method for the analysis of very small concentrations (thousandths of one per cent) of boron in steel.

Filings, taken from several parts of the sample with a very fine file, are sifted through a fine screen with holes 0.3 mm in diameter. The undersize is then carefully mixed, and a weighed portion of 200 mg is taken. The latter is placed in a shallow hole drilled in an aluminium block. An activized alternating-current arc burning between the sample as one of the electrodes, and an Armco iron rod as the other, is used as the excitation source for the spectrum. The crater of the

arc does not touch the block, but just moves from place to place over the surface of the filings, burning them rapidly and thus giving rise to very intensive evaporation.

By means of this method we have been able to achieve extremely high sensitivity in our boron determinations. An exposure of 30 sec. was found sufficient to bring the $B\ \lambda = 2496.7$ Å line to a normal state of blackness with the boron concentration only 0.003 per cent, the constant spectrum background being practically absent.

A comparison of the spectra obtained from a solid sample and from a powder prepared from it, the exposure being equal in both cases, showed that the boron lines in the spectrum of the powder were 10—15 times brighter than in that of the solid sample. Experiments with Si and Mn showed that the lines of these elements are also much brighter in the spectra of powdered samples.



The problem of preparing standards for use in this method is also solved very simply. They can be prepared by «diluting» powders made from samples with intermediate concentrations of the element to be determined. For instance, in our boron analyses we prepared our standards from a sample containing 0.10 per cent boron. The sample was filed and the filings were sifted, after which Armco iron filings were added in the required proportions. The filings were carefully mixed in order to distribute those containing the boron evenly throughout the whole sample. A calibration graph plotted for standards prepared in this manner, and containing 0.005, 0.012, 0.02 and 0.05 per cent boron, is given as an example in the figure.

It stands to reason, of course, that standards prepared in this manner may not quite correspond in structure to the samples analysed, which may result in systematic errors in the analysis. It seems to us, however, that this danger can hardly be a serious one, since the influence of such factors in the arc is insignificant, and besides, the filings soon become fused together, so that the differences in structure of the metals are soon blotted out. Finally, even if we assumed systematic errors to be possible, they would never exceed 10—20 per cent of the total content determined, which, considering that this total content is of the order of only several thousandths of one per cent, is of no practical significance.

Coming back to the intensification of spectral lines observed when working with powders, it must be noted that all lines are not intensified equally well. Some of the lines of iron and of the admixture elements added to it, as has already been pointed out, are greatly intensified, others practically undergo no change, while still others are weaker in the spectrum of the powder than in that of the solid electrodes. When the lines were deciphered, it was found that those which were more intense in the spectrum of the powder were the arc lines, and those that were weaker—the spark lines of the corresponding elements. Besides, when the arc is stigmatically reproduced in the slit of the spectrograph, the line in

tensities in the spectrum of the solid electrodes are distributed differently from end to end of the arc than in the spectrum of the powders; in this case there is also a difference in the behaviour of arc lines and spark lines.

In concluding, the author takes the opportunity to express the deep gratitude due to Professor S. L. Mandelstam, under whose supervision this work has been carried out.

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PHYSICAL CHEMISTRY

ANALYSIS OF THE CRYSTAL STRUCTURE SiCV (51-LAYERED PACKING)

By G. S. ZHDANOV and Z. V. MINERVINA

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1. Literature Data. The structures SiCI, SiCII, SiCIII and SiCIV were identified by Ott([1]), who besides this published the data of an X-ray study of a very rare silicon carbide crystal, which he specified as SiCV[2]. This SiCV crystal, in the shape of a thin plate 1×1.5 cm was found among the crystals of the other known modifications of silicon carbide on a large piece of melt. This modification of silicon carbide is distinguished from the others by the unusually large period along its axis $c = 128$ Å. The size of the period in the plane of the base (a 3.08 Å) does not depend upon the type of the modification. The periodicity of the structure along the c-axis, n_c, is 51 (!). In the rhombohedral axes $a_r = 42.8$ Å, and $\alpha = 4°8'$. The minimum periodicity along the edge of the rhombohedron is $n = 17$, *i. e.* $3^2/_5$ times that in SiCI and $2^5/_6$ times that in SiCII. The position of the atoms in the SiCV structure has not been established owing to the large number of possible arrangements (some 30 000 structures, according to Ott) and to the fact that it is practically impossible to compute the intensities for all of them.

In our analysis of the SiCV structure we employed Belov's theory of the close packings of spheres. A new conception introduced by us—a numeral symbol for close packings of spheres ([5])—has enabled us to establish the relationships in the structures of silicon carbide already known, and on the basis of this to propose several probable models for the SiCV structure. Calculated intensities agreed excellently for one of the probable variants with the experimental data obtained by Ott.

2. Numeral Symbols for the Known Structures of Silicon Carbide. Silicon carbide crystals have diamond-like or tetrahedral lattices, in which each A atom (silicon) lies within a tetrahedron of B atoms (carbon), and *vice versa*. The A and B lattices taken separately are exactly alike, and belong to one of the types of close packings, so that they may be characterized by a numeral symbol. We shall regard this numeral symbol also as fully describing the tetrahedral structure of silicon carbide. The coordinates of the A atoms are found from the numeral symbol by a method described elsewhere([5]). The coordinates of the B atoms are obtained from the coordinates of the A atoms by adding to all the z coordinates the lengths of the displacement vectors of both lattices $z_s = {}^3/_4 n_c = {}^1/_4 n$.

The numeral symbols as given in Table 2 of Zhdanov's paper([4]) for silicon carbide lattices with one main axis are: SiCII—3.3; SiCIII—2.2; SiCI—2.3. These three numeral symbols have one feature in common: they are made up only of twos and threes*. The absence of packings among silicon carbide crystals with layers consisting of one elementary layer each, such as the würzite packing (1.1), and other simple packings, for example, 1.2, 1.3, *etc.*, is striking. Judging from

* Our values for the periods are corrected. The value of the periods $a = 3.09$ Å given by Ott in all his works, is nearly 0.02 Å higher than those ($a = 3.076$ Å) found in subsequent papers. Hence $c = 0.817 \cdot 51$, $a \cong 128$ Å.

the data in literature and from the results of X-ray phase analyses of various silicon carbide made in our laboratory, the most prevalent modification is SiCII, built up of triple layers.

3. Numeral Symbols for the Probable Packings of SiCV. Let us assume that SiCV is also built up of double and triple layers. The construction of corresponding models of 17-layered structures is equivalent to finding their numeral symbols, and this reduces to the simple arithmetical problem of finding all the ways of resolving $n=17$ into an even number m of twos and threes. This problem has only two solutions, *viz.* $m=6$ and $m=8$. Variant I—2.3.3.3.3.3 and variant II—2.2.2.2.2.2.2.3. These numeral symbols show the probable packings of SiCV. Both packings are rhombohedral and of similar symmetry: D_{3d}^5-R3m.

The tetrahedral lattices corresponding to these symmetries are polar and belong to the space group C_{3v}^5-R3m. In order to find the correct variant of the SiCV structure and verify it, it is necessary to compute the intensities of its X-ray interference patterns.

4. Structural Amplitude of SiCV. According to the theorem of the structural amplitude of a complex lattice, the structural amplitude of a binary tetrahedral structure with a rhombohedral Bravais lattice may be written (in hexagonal axes) as follows: $F(hkl)=F_rF_AF_z$ where F_r is the amplitude of the base of the rhombohedral lattice; F_A, the amplitude of a close packing of spheres, and F_Z, the amplitude of shift of the lattices of carbon and silicon in relation to one another.

$$F_A=\sum_p e^{2\pi i\frac{pl}{n_c}}+\sum_q e^{2\pi i\left(\frac{2}{3}+\frac{q}{n_c}\right)l}+\sum_r e^{2\pi i\left(\frac{1}{3}+\frac{r}{n_c}\right)l} \quad \text{and} \quad F_z=f_{Si}+f_Ce^{2\pi i\frac{l}{4n}}$$

where f_{Si} and f_C are the atom factors of silicon and carbon atoms, respectively.

The parameters p, q and r, which determine the arrangement of the atoms in a packing of spheres can be calculated from the numeral symbols and have been found to have the following values:

Variant I: $p=0, 4, 6, 10, 12, 16$; $q=1, 3, 7, 9, 13, 15$; $r=2, 5, 8, 11, 14$.
Variant II: $p=0, 4, 8, 12, 16$; $q=1, 3, 5, 7, 9, 11, 13, 15$; $r=2, 6, 10, 14$.

If $h-k=3t$, it follows from the condition for a rhombohedron $h-k+l=3t$ that l will be a multiple of $3t$. From the formula for the structural amplitude of a packing of spheres we get

$$F_A=\sum_{p=1}^{n} e^{2\pi i\frac{pt}{n}}=\begin{cases}0 & \text{when } t\neq nu\\ n & \text{when } t=nu\end{cases} \quad (u=0, 1, 2, \ldots.)$$

If $h-k\neq 3t$, then from the condition for a rhombohedron we get $F_r=0$ if $l=3t$. If, on the other hand, $l\neq 3t$, then $F_A\neq 0$, $F_r\neq 0$ and $F(hkl)\neq 0$.

5. Computation of $|F_A|^2$ and Comparison with Experimental Data. The structural amplitude of a tetrahedral packing is a function only of Miller's index $F(hkl)=F(l)$; this considerably simplifies the computation of the intensities. For a packing of spheres $|F_A(l_1)|^2=|F_A(l_2)|^2$, if $|l_1|+|l_2|=n$. Therefore it is sufficient to compute $|F_A|^2$ for only $\frac{1}{3}n_c=n$ interferences. *i. e.* for 17 independent points in the case under consideration. Calculated values of $|F_A|^2$ for both variants are given in the accompanying table.

The values of $|F_A|^2$ were compared with experimental intensities determined visually from reproductions of the oscillations X-ray photographs of SiCV as

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* Recently Thibault (4) has found new silicon carbide structures, which he denotes as SiCIV and SiCVI. The numeral symbols of the SiC VI—2.3.3.3. and of the SiC IV—3.4.

Calculated Values of $|FA|^2$ for the Two Variants of the SiCV Structure and Experimental Values of I

l_1	l_2	$\|FA\|^2$		I exp.			
				21 l		10 l	
		variant I	variant II	l_1	l_2	l_1	l_2
1	$\overline{50}$	0.3	10	0	—	v. w.	—
2	49	0.3	5.1	0	—	v. w.	—
4	47	0.6	0.3	v. v. w.	—	v. w.	—
5	46	0.5	1.9	v. v. w.	—	w.	—
7	$\overline{44}$	7.4	0	m.	w.	m.	—
8	43	22 3	2.6	s.	s.	s.	—
10	$\overline{41}$	14.7	0.1	m. s.	w.	m. s.	—
11	40	0.9	14.7	v. v. w.	0	w.	—
13	$\overline{38}$	5.8	59.3	w.	v. v. w.	m.	—
$\overline{14}$	37	0.2	31.5	0	0	v. v. w.	—
16	35	51.7	1.2	v. s.	s.	s.	v. s.
$\overline{17}$	34	64.0	4.0	v. s.	v. s.	v. s.	v. s.
19	$\overline{32}$	14.5	16.2	m. s.	v. w.	m. s.	s.
$\overline{20}$	31	0.4	3.1	v. v. w.	0	v. w.	0
22	$\overline{29}$	7.1	2.9	w.	v. v. w.	m.	m. s.
$\overline{23}$	28	2.2	7.7	0	0	m. s.	w.
25	$\overline{26}$	98	143.1	v. s.	v. s.	v. s.	v. s.

Note. v. v. w. — very very weak, v. w. — very weak, w. — weak, m. — medium, m. s. — madium strong, s. — strong, v. s. — very strong.

cited by Ott(2). In computing the intensities no account was taken of the shift factor $|F_Z|^2$, which is a monotonic function of l and is compensated to a considerable degree by the kinematic factor, first introduced by Ott. No account was taken of the other monotonic intensity factors, inasmuch as the variants of the structure under consideration differ in the values of their parameters which vary discretely, thus eliminating the necessity of taking close values of the parameters and of making precise measurements of the intensities.

A comparison of the calculated and experimental values of the intensities excludes variant II and confirms the correctness of variant I, for which excellent agreement between calculated and experimental results is obtained. Variant I—2.3.3.3.3.3—is a «slightly spoiled» modification of SiCII: 2.3.3.3.3.3 = 17 = 18—1 = 3.3.3.3.3.3—1.

Conclusion. With the aid of numeral symbols for close packings of spheres the relationships in silicon carbide structures built up of double and triple layers have been established. The assumption that the structure SiCV is analogous to a 17-layered packing allows for two probable models of the structure, with experiment unambiguously pointing to the structure with the packing symbol 2.3.3.3.3.3, which is the closest to the main modification SiCII whose packing symbol is 3.3.

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PHYSICAL CHEMISTRY

THE STRUCTURE OF HEAT-RESISTANT AUSTENITIC STEEL

THE KINETICS OF GRAIN GROWTH AND THE SOLUTION OF CARBIDES IN EI-69 STEEL UPON HEATING

By Z. L. MINZ and G. V. AKIMOV, Corresponding Member of the Academy

Austenitic steel containing 13 Cr, 13 Ni and 2W and sometimes up to 2 per cent Si and 0.5 Mo as well (EI-69) is one of the most prevalent high-strength heat-resistant steels. Such steels are used all over the world to manufacture valves for high-power aircraft motors, turbine blades and a number of other vital parts. The heat resistance of this type of steel can be increased appreciably by tempering at a temperature of 1200—1250° C, instead of the usual 1050—1100°. This results in a considerable increase in the size of the austenite polyhedra in the steel, making its structure coarse-grained. The increase in heat resistance is accompanied by a pronounced decrease in plasticity ([1–2]). In practice the heat resistance of the parts can be increased from two to fourfold by raising the tempering temperature, but at the same time the elongation decreases by 50—80 per cent.

In order to establish the causes of such a sharp change in the mechanical properties at high temperatures, it is necessary, first of all, to make a study of the kinetics of grain growth and the solution of carbides in EI-69 steel during the heat traetment used in tempering. In our experiments the steel specimens were quenched in water after being held at temperatures from 900 to 1300° for periods of 5 min. to 75 hrs. The number of carbides and their size were determined microscopically. Grain size in specimens tempered at temperatures of 1200° and higher was also determined under the microscope.

Below 1200° grain growth was established indirectly, by means of the X-ray method, from the number of interference spots. The large number of carbides left undissolved makes microscopic determination impossible in these cases. It was found that even at 900° a clearly detectable, though slow, growth is also observed in the austenite grains.

Fig. 1 shows the relation between the grain growth and the temperature and duration of heating. This figure shows that grain growth begins to accelerate pronouncedly between 1200—1250°. This corresponds to a practically complete solution of the carbides in the steel. The kinetics of carbide solution is pictured in Fig. 2. From this diagram it follows that as the temperature rises, the carbides dissolve continuously more and more. For all temperatures, except 900°, increasing the duration of heating results in the number of visible carbides decreasing.

The relation between the number of carbides and the duration of heating at 900° may be represented by a curve with a slight maximum. Thus, during many periods of heat treatment the density of the carbides (the number of carbides per

unit volume) first rises and then slowly falls. This is due to several different processes taking place simultaneously: a) solution of the carbides in the alloy, b) coagulation of microscopically distinct carbides into larger ones, c) coagulation of microscopically invisible carbides to dimensions detectible under the microscope.

Processes a) and b) decrease the number of carbides detectible under the microscope, while process c) increases the number of visible carbides.

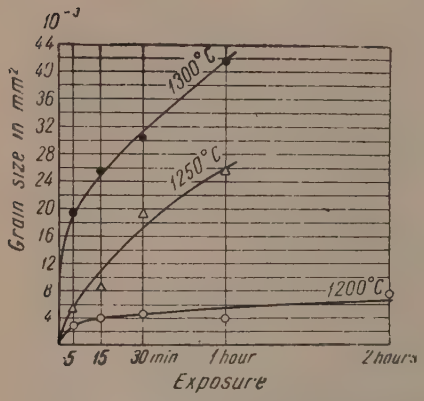

Fig. 1. Relation between the grain size of E1-69 steel and the temperature and duration of heating in tempering

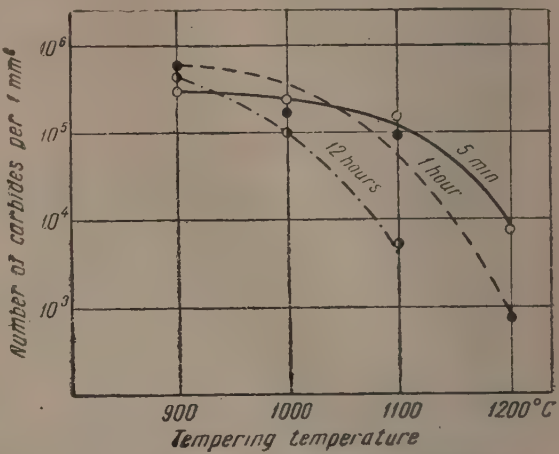

Fig. 2 Relation between the number of carbides in E1-69 steel and the temperature and duration of heating in tempering

Thus, for short periods of heating at 900° C, the coagulation of microscopically invisible carbides predominates, but as the heating period is increased, the two other processes gradually begin to predominate. At other temperatures the solution and coagulation of microscopically detectible carbides predominate. For very long periods of heating (50 hrs.) the carbides coagulate into extremely large particles, 20—100 times coarser than the original carbides in the steel.

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PHYSICAL CHEMISTRY

CONTRIBUTION TO THE COMPUTATION OF THE CHARACTERISTICS OF A DETONATION WAVE PROPAGATED AS A STATIONARY WAVE THROUGH CONDENSED EXPLOSIVES AND THE EQUATION OF STATE FOR THE PRODUCTS OF EXPLOSION

By S. B. RATNER

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1. For the determination of the four characteristics of a detonation wave—the velocity of detonation (D), the velocity of the explosion products immediately behind the front (U), their density ($\rho = 1/v$) and the pressure in the front (p)—the hydrodynamic theory of detonation provides us with the three conservation laws (mass, momentum and energy) and the Jouguet-Cruyssar condition:

$$D = U + C = v_0 \sqrt{-(\partial p/\partial v)_s} \qquad (1)$$

which says that the velocity of detonation is equal to the relative velocity of sound (C) in relation to that of the explosion products. Without going into the mechanism of the detonation of condensed explosives, we shall assume condition (1) to be applicable. As to the applicability of the conservation laws, no reservations are needed.

The introduction of the equation of state for the explosion products, without which the isoentropic derivative of pressure with respect to volume in condition (1) cannot be determined, enables us to find a fifth characteristic, *viz.* the temperature of detonation (T). Application of the Klapeyron equation to the explosion products gives:

$$D_{\text{ideal}} = \frac{k+1}{\sqrt{k}} \sqrt{\frac{n}{M} R} \sqrt{T} \cong \frac{\sqrt{T}}{20} \text{ (km/sec.)} \qquad (2)$$

since $k = c_p/c_v \cong 1.2$, and M/n, the mean molecular weight of the explosion products (consisting chiefly of CO_2, H_2O, N_2 and CO for many explosives), may be assumed equal to 30 g. Inasmuch as ordinarily $T = 3000—5000°$, the detonation velocity is close to 2 km/sec.

From (2) it follows that this detonation velocity does not depend on the density. Actually, however, it is observed to increase with the latter and may reach 8 km/sec. This is due to the fact that its value is affected considerably by the non-ideal nature of the explosion products, whose density reaches 2 g/cm³.

For liquid and solid explosives the computation can be carried out by using an equation of state that correctly describes the properties of the explosion products. In this case all the formulae connecting the parameters of the detonation wave with the temperature are modified.

2. One of the first attempts to perform such a computation (in application to nitroglycerine and mercury fulminate) was made by Becker in 1916(1). He constructed an equation of state for nitrogen

$$p = RT\rho\,[1 + f\rho e^{f\rho}] + a\rho^2\,[h\rho^s - 1] \qquad (3)$$

which is in accordance with the experimental data obtained by Bridgman (2) at 65°C for densities up to 1.1 g/cm³*. This specific equation cannot be employed, however, since we have no right to identify the explosion products with nitrogen, especially in view of their higher density.

It is evidently due to these circumstances, and to the fact that expression (3) complicates the mathematics that Becker's work has failed to find adherents in literature.

3. Considerably more progress has been made in the opposite line of approach chosen somewhat earlier (1912) by Taffanel and Dautriche (3), which found support in the works of Jouguet (4) and later (1935—1936) in those of Schmidt (5).

These investigators assume the equation of state to have the general form

$$p(v - \alpha) = \frac{n}{M} RT \qquad (4)$$

and assuming the co-volume α to be constant, deduce the following equations** on the basis of the hydrodynamic theory:

$$D = \frac{k+1}{\sqrt{k}} \sqrt{\frac{n}{M} RT} \frac{1}{1 - \alpha\rho_0} = D_{\text{ideal}} \frac{v}{v_0 - \alpha} \qquad (5)$$

Since D_{ideal} can be calculated from formula (2), these authors propose calculating α from experimental data for the relation between D and the initial density $\rho_0 = 1/v_0$, and then using it for interpolation and extrapolation, and in calculating the detonation velocities of other explosives (5).

However, calculations carried out for dynamite, TNT, PETN and other explosives (5) showed that the co-volume is not constant, but increases with the specific volume. Hence it is incorrect to treat α as a constant when substituting (4) into (1)*** for the detonation of condensed explosives, so that the formulae given by Taffanel, Dautriche and Schmidt are untrue.

4. In order to bring Taffanel and Dautriche's idea to life correctly, it is necessary to select such a form for the equation of state of the explosion products that will fit in with experimental data. With this end in view we made use of Bridgman's data (2) mentioned in paragraph 2, above. They indicate that equation (4) may be applied within the density range he investigated, if the co-volume is expressed as a symbatic function of the volume. In doing this, we accepted the Van Laar form of this function $\alpha = bv/(c + v)$, as a result of which equation (4) becomes:

$$p = \frac{n}{M} RT \frac{v + c}{v\,[v - (b - c)]} \qquad (6)$$

where $b > c > 0$. It is these latter constants that have to be determined from experimental data for $D = D(\rho_0)$.

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* It should be noted that the part of formula (3), which does not depend upon the temperature, contains a positive term connected with the forces of repulsion, which exceeds the temperature term at densities close to 2 g/cm³. Here and henceforth we refer to the elastic forces of repulsion against which work is done during compression, so that they affect $(dE/dp)_T$. They differ essentially from the forces of repulsion usually associated with the co-volume. Although the latter are taken into account in formulae (4) and (6), this does not change the expression $(\partial E/\partial p)_T$, which remains equal to zero just as in the Klapeyron equation.

** A formula analogous to (5) was obtained as far back as 1891 by Vieille(6), who used equation (4) in the Laplace formula showing the relation between the elasticity of the gas and the velocity of sound, which he equated to the velocity detonation.

*** Our attention was drawn to this error by Professor J. B. Zeldowich.

5. The introduction of formula (6) into the fundamental equations gives rise to relations quite different from the foregoing. For U we get:

$$U = \sqrt{\frac{nRT}{Mk\left[1 - \frac{bc}{k(v+c)^2}\right]}} = \frac{U_{\alpha=\text{const}}}{\sqrt{1 - \frac{bc}{k(v+c)^2}}} \tag{7}$$

where $U_{\alpha=\text{const}}$ is in conformity with paragraph 4.

A system of two equations takes the place of formula (5).

$$D = D_{\text{ideal}} \frac{\sqrt{k}}{k+1} \frac{x}{\sqrt{y}\,(v-b+c)} \tag{8}$$

$$\rho_0 = \rho \frac{x}{y} \tag{9}$$

where $x = (k+1)c^2 + [2(k+1)c - b]v + [(k+1)c - 2b]c$, and $y = kv^2 + 2kcv + (kc - b)c$.

The algebraic solution of this system is very cumbersome (taking into account the expressions for x and y), although, principally, it gives a very simple relation between D and ρ_0 (through the parameter v). This is very important, because the relation between them can be verified by experiment.

6. In order to show the magnitude of the deviations which arise from Schmidt's error, we took his initial data for PETN: $k = 1.2$; $M/n = 29.2$ g; $T = 5000$ K; $D_{\text{ideal}} = 2.4$ km/sec., and the results of Friedrich's experiments on $D = D(\rho_0)$ that he used. Solving the system of equations (8—9) graphically, we got $b = 1.6$ cm³/g; $c = 1.36$ cm³/g. Hence the equation of state for the explosion products of PETN is

$$p\,(\text{atm}) = 83.1 \frac{n}{M} \frac{v + 1.36}{v\,(v - 0.24)} T \tag{10}$$

As a result, instead of the constant value of 1.09 km/sec. obtained for U by Schmidt, we found that within the range $0.25 - 1.6$ g/cm³ it increased progressively from 1.16 to 1.6 km/sec. For the pressure in the front our results are proportionally higher than those obtained by Schmidt (for example, at $\rho_0 = 1.2$ Schmidt reports a pressure of 82 000 atm.; our value is 114 000 atm.). The same holds for the other parameters as well.

7. In order to draw conclusions as to the range of applicability of the formulae we have obtained, it is necessary to compare the results of calculations made from them with direct experimental data (Bridgman). Such a comparison (for nitrogen) is given in the accompanying table.

ρ, cm³/g	$p \cdot 10^{-3}$ atm.	
	experiment after Bridgman [2]	calculated by formula (10)
1.29	3	1.97
1.09	6	2.63
0.98	10	3.22
0.91	15	3.75

The nature of the discrepancies between these figures confirms the growing rôle of the forces of repulsion, not depending on the temperature, as the density rises, which has already been taken into account in Becker's formula (3). Thus, the form of the equation of state adopted in our computations, $p = RT\varphi(\rho)$, cannot reflect the properties of the explosion products if the initial density of the explosive is high.

8. At the same time the use of precisely this form opened up prospects of effecting a complete computation of the characteristics of the detonation wave in

condensed explosives, since it bridged the gap between them and the gases which present no unsurmountable difficulties for calculations, as Zeldovich and the author have shown (7).

Hence it follows that not only are the formulae and computations, given by Taffanel, Dautriche and Schmidt, incorrect, but even their idea of using a correction coefficient D/D_{ideal} to offset the unideal nature of the explosion products cannot be put into effect.

To find the correct relation between the quantities characterizing the detonation wave, an equation of state that takes into account the forces of repulsion must be utilized*.

9. At the same time, inasmuch as for definite classes of explosives the explosion products are also close in composition, it may be assumed that within these classes that part of the pressure which depends upon the temperature will not change much at a given density. This makes it possible, for example, to apply the methods of computation developed here to liquids.

Conclusions. 1. A criticism is given of the preceding works on this subject: those of Becker, Taffanel and Dautriche, Jouget and Schmidt, on the basis of the general equations of the hydrodynamic theory of detonation.

2. The equations of the hydrodynamic theory are solved by using the van der Waals equation of state, and expressing the co-volume in it in the form proposed by van Laar.

3. The constants of the equation of state for the explosion products have been calculated from experimental data for the relation between the velocity of detonation of PETN and its density.

4. Comparison with the experimental data given by Bridgman indicates that at high densities it is necessary to take into account forces of repulsion, not independent the temperature.

The author takes the opportunity to express his gratitude to J. B. Chariton and J. B. Zeldovich for their helpful discussion in the course of this work.

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* Our attention was drawn to this circumstance by Professor L. D. Landau

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GEOCHEMISTRY

POTASSIUM SALTS OF THE INDER UPLIFT

By J. J. YARZHEMSKY

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Potassium was first discovered in the Inder area by chemical analyses of the brine of the lake, after the presence of potassium compounds was suspected in the salts of the uplift, mineralized waters being supplied to the depression of Lake Inder at the expense of the erosion of the upper parts of the uplift. In this connexion, during 1927—1931 the Central Scientific Institute for Geological Survey (ZNIGRI) undertook prospecting which resulted in 1934 in the discovery of borates. No conclusive data as to the occurrence of potassium in the Inder uplift were, however, obtained.

Since 1935—1936 the Inder area has become a large-scale producer of borate raw material. Systematic geological prospecting carried on within the Inder uplift resulted in the discovery of a number of borate occurrences in the rocks of the gypsum cap and of potassium compounds in its salts. Some of such potassium salts have been found to be high in boron. Owing to this, these horizons could be considered as a source of supply of both potassium and boron. Since then the Inder area has assumed an important place not only as a source of boron ores, but also of potassium, highly valuable for the national economy.

As a result of geological prospecting work carried on by several organizations (the All-Union Geological Institute, the Institute for Mining and Chemical Raw Material (GIGKhS), *etc.*) in the Inder area five potassium-bearing deposits have been found in the upper portion of a thick uplift composed chiefly of halite rocks.

Of particular interest are the results of drilling carried out within No. 1 potassium deposit located 14 km east of «Inderborstroy», between Nos. 7 and 8 borate areas. Here the Kazakhstan Geological Administration drilled two profiles. Highly valuable materials, presented by A. N. Volkov in his report on the work in the Inder area in 1940, were obtained from the data of the 1st prospecting profile. In the section of hole No. 238 sylvites begin directly from the surface of the salt table. At a depth of 49.8 to 168.3 m, 118.5 m thick along the vertical, nearly uniform sylvites are penetrated with an average potassium content (from 29 samples) (K, and not K_2O) of 25.9 per cent. This figure is exceptionally high, and the occurrence is noteworthy. The value of the deposit is still further increased by the fact that the occurrence of potassium salts continues still deeper. From 168.3 to 181.15 m follows an interbedding of sealing wax-red sylvite with thin layers of polyhalite and halite. Within this interval the average potassium content amounts to 20.83 per cent. From 181.15 to 199.78 m follows halite interbedded with polyhalite. From the depth of 199.78 m to the face of the well (to 220.47 m and deeper) follows spotty-coloured sylvites similar to those occurring at the upper horizons, but somewhat poorer in potassium (on the average 18 per cent).

Later on (in 1941—44) GIGKhS drilled an intermediate profile between those of Kazgeological Administration (1st and 2nd). This intermediate profile permitted an estimation of the reserves of potassium salts in the south-eastern half of the deposits to be made, and the point of sinking a prospecting and productive shaft to be determined on No. 1 potassium deposit.

The second potassium deposit has been opened up only by three closely spaced drill holes by the author within the salts underlying the borate giant of caprock No. 36.

It is of great interest, owing to the fact that here for the first time in the Inder area kainite salts of economic value have been discovered in paragenesis with primary borate (kaliborate) ores. These deposits require further study, as well as potassium deposit No. 3 (at borate occurrence No. 31a) composed of carnallites, discovered in the course of the previous prospecting work of Kazgeological Administration.

The work done has revealed a highly complex mode of occurrence of the potassium salts in the Inder area, an absence of any reliable marking elements, which greatly impedes a detailed study of the deposits by means of core drilling with the object of establishing the reserves of workable categories. The latter can be determined only in the course of underground prospecting and mining which should be undertaken in the near future for a further study of the deposit.

The potassium salts of the Inder uplift rank second in the USSR after Solikamsk with its inexhaustible reserves. While the reserves of potassium salts in the Inder are smaller than those of Solikamsk, the Inder area possesses even some advantage over the unrivalled potassium giant of the world.

While Solikamsk deposits contain only potassium chlorides, being characterized by the absence of sulphates, the Inder deposits, along with sylvites, carry also potassium sulphates (polygalites, kainites). The sylvites of the Inder area are, as a rule, higher in potassium than those of Solikamsk and of the near-Carpatian deposits (Kalusha, Stebnika). The potassium salts of the Inder area are as yet the only ones in USSR which not infrequently occur together with parent borates, often of an economic value or as a small admixture, which adds to the potassium salts a valuable admixture of boron microfertilizers. The promising boron resources of the Inder area (as yet the only producer of boron for our domestic industry) are intimately coordinated with the potassium problem.

The high potassium content of sylvites in No. 1 deposit within the 1st profile of Kazgeological Administration secures the production of fertilizing salts by merely grinding raw sylvites without adding any high-grade concentrated commercial potassium chloride. Ground sylvite with a potassium content of 25 per cent should give a fertilizing salt of commercial grade with 30 per cent of salts (30 per cent K_2O or 47.5 per cent KCl).

As seen from literature ([1], [2]) when using raw potassium salts as fertilizers they are found favourably to compare with the concentrated ones.

In this way, geological and prospecting work done within the recent 5—7 years has revealed in the Inder area the existence of a highly valuable source of potassium and boron which will supply the socialist agriculture of Central Asia, near-Caspian area and North Caucasus with mineral fertilizers and boron raw materials, necessary for the industry of our country.

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GEOLOGY

A CONTRIBUTION TO THE GEOCHEMISTRY OF HYDROGEN SULPHIDE OF THE NEAR-VOLGA ZONE OF THE SVIYAGA-SURA DIVIDE

By I. K. ILLARIONOV

(Communicated by P. I. Stepanov, Member of the Academy, 11. V. 1945)

The divide between the rivers Sura and Sviyaga stretches very far South-East of the Volga. When speaking of the near-Volga zone we mean the basins of the Big and Little Tzivil Rivers, the basins of the left-hand side tributaries of the Sviyaga from its mouth to the Karla River inclusive, and the basins of the right-hand side tributaries of the Sura from its mouth to the Bezdna River inclusive. The geological structure is represented by the following formations which either outcrop on the surface, or have been opened up by deep drill-holes to a depth of 200—250 m from conventional zero*.

Of the oldest formations thus far Kungurian deposits are known, represented by bluish anhydrites, interbedding with thin layers of dolomites. The thickness exposed is 15—20 m.

These deposits are underlain by a series of Kazan dolomites, at the base of which occasionally occur dolomitized limestones. In some areas they are more or less bituminized. The base of the carbonate series of the Kazan stage usually terminates with a layer of gypsum 8—10 m, and in some places 12 m thick. The base of the whole series of rocks of the Kazan stage begins with layers of anhydrites, 20—25 m thick, interbedding with layers of clayey dolomites.

At the base of dolomitized limestones there is scarcely any gypsum if at all, and in the roof of the Kazan deposits it is present only at those places where these deposits lie at comparatively low hypsometric marks. The total thickness of the carbonate rocks of the Kazan stage is 60—90 m, and if taken together with the sulphate layers the Kazan deposits attain 18—115 m in thickness. In the system of the Volga and Sura Rivers, within the above-mentioned part of the divide, no Belebeyevo deposits are present.

Higher up follow the rocks of the Tartar stage, represented by variegated formations. Both the Urzhum and the Sarma series are present. In the area of the eastern half of the near-Volga zone of the Sviyaga-Sura divide the Sarma series is not fully represented.

As we ascend towards the upper boundary of the Tartar stage, the quantity of magnesia strongly diminishes, and that of dark minerals increases, both in carbonate and in other rocks.

The total thickness of the Tartar deposits in the eastern half of the above-named divide is 200—225 m, and in the western half, 300—315 m.

* As conventional zero, the section of the Volga at the town of Cheboksary has been taken; it is located approximately between the mouths of the Sura and Sviyaga Rivers.

Mesozoic deposits are represented by comparatively uniform dark-grey and dark clays of the Upper Jurassic and Lower Cretaceous, but the Upper Cretaceous deposits are strikingly different from the rest, being represented by light marlaceous tripoli. At the contact between Jurassic and Cretaceous deposits there is a productive phosphorite horizon, underlain by oil shales of the *Pavlowia panderi* zone: the thickness of these shales is 0.5—2.25 m. A certain amount of phosphorites in the shape of basal conglomerate is present also at the base of Upper Cretaceous tripoli (Turonian).

Tertiary deposits have been found only at some points in the basin of the Sura and the middle course of the B. Tzivil. They are represented by detached spots of quartz sands with pebbles and breccia-like fragments of gaizes. Quaternary deposits occur in the shape of a certain cover of post-Pliocene loams or sands both over Palaeozoic and over Mesozoic deposits, the sands being most wide-spread in the area of development of Mesozoic deposits. In diluvial loams rather often incisors and parts of the skeleton of the hairy rhynoceros are encountered. In some places the author has discovered glacial boulders and remains of a strongly decomposed moraine.

In the light of geological and geochemical analysis of great importance are the cases of hydrogen sulphide issuance established by the author at a number of points within the near-Volga zone of the Sviyaga-Sura divide. In most cases hydrogen sulphide was discovered when drilling deep holes at a depth of 200—250 m from the surface, the output of H_2S markedly increasing with depht. Hydrogen sulphide discharges at the day's surface have been also recorded. Such cases have been reported from the system of the L. Tzivil, B. Tzivil and Unga rivers, and at other places. Where hydrogen sulphide issues at the day's surface, the area is strongly dislocated.

Owing to the fact that almost everywhere hydrogen sulphide has been met at a comparatively large depht, and with increasing depht of the face of the well the discharge is markedly increasing, issuing at the day's surface in dislocated areas,—the possibility of its superficial origin may be fully discarded.

In the author's opinion, under the geological conditions of this part of the Russian platform the geochemistry of hydrogen sulphide from deep horizons should be accounted for chiefly by the presence of gas and oil pools at depth.

Within the near-Volga zone of the Sviyaga—Sura divide no oil showings have been as yet discovered directly at the surface, neither its oxidized remains in the shape of asphalts, brittle bitumens or other compounds have been encountered. When sinking deep wells in the region of Maryinsky Possad, Tzygan-Kassa, Cheboksary, Ilyinka and at some other places, in the rocks composing the base and the roof of Kungurian deposits, as well as in the waters of these wells, when opening up the above-named horizons, a rather strong smell of oil was perceived for the first time. Moreover, a showing of oil at the surface was established in 1939—1941 in the immediate vicinity of the northern and of the above divide, namely in the region of the Dubovka wharf, on the left bank of the Volga.

Certain amounts of hydrogen sulphide are known to accumulate in the Earth's crust in oil-bearing horizons; under conditions favourable for migration, it ascends to higher lying horizons, together with other vapours and gases. Aside from this, in particular under the conditions of the Volga region, the accumulation of considerable masses of hydrogen sulphide at comparatively large depths should be accounted for, in the author's opinion, by processes of reduction of sulphates by liquid hydrocarbons or their vapour. Within the areas of the near-Volga zone of the Sviyaga-Sura divide liquid hydrocarbons and their vapours may have penetrated to the sulphate horizons, as a result of tectogenesis taking place subsequently to the formation of sulphate horizons.

Different opinions exist as to the reduction of calcium sulphates by bitumens and the ultimate product of this reaction. Some investigators [2,4] discard the possibility of a reduction of calcium sulphates by bitumens under natural conditions, according to the formula:

$$R''SO_4 + 2C = R''S + 2CO_2$$

or

$$CaSO_4 + CH_4 = CaCO_3 + H_2O + H_2S$$

owing to the alleged endothermal nature of this reaction. In the opinion of Danilchenko, Chighirin and Berg([1,2]), if the possibility of a reduction of calcium sulphates by bitumens be admitted, the course of the reaction would have been some what different, free sulphur instead of hydrogen sulphide being subsequently formed. Thus, for instance, after Danilchenko and Chighirin, the reaction should proceed with a gradual formation of intermediate oxides, free sulphur inclusive:

$$SO_4'' \rightarrow SO_3 - S_2O_3 \rightarrow S$$

or, after Berg ([1])

$$CaSO_4 + 2C + H_2O = Ca(SH)_2 + CaCO_3 + CO_3$$
$$5CO_2 + 3Ca(SH)_2 + 2CaSO_4 + 3H_2O = 5CaCO_3 + 8S + 6H_2O$$

Other investigators consider it necessary for the processes of reduction of sulphates by bitumens the presence of more complicated high-molecular bitumens and of good catalysts.

The author believes that processes of reduction of sulphates, with subsequent formation of hydrogen sulphide do really take place in nature, and to make them possible there is no need in the presence of complex high-molecular bitumens and catalysts. This is supported by the results of the experimental studies of Sakhanov and Luchinsky who proved the exothermal nature of the reaction of reduction of calium sulphates([5]). The author thinks that in a simplified way this process will take place according to the following reaction.

$$CaSO_4 \cdot 2H_2O + CH_4 + nH_2O = CaCO_3 + nH_2O + H_2S$$

or

$$CaSO_4 + CH_4 + nH_2O = CaCO_3 + nH_2O + H_2S$$

In the bowels of the earth a reduction of sulphates with a subsequent formation of hydrogen sulphide undoubtedly takes place also as a result of biochemical processes. In oil-bearing beds there live anaerobic bacteria which reduce the sulphates at the places of the contact of the latter with bitumens, considerable masses of hydrogen sulphide being formed.

Now let us discuss the question as to the conditions and time of the formation of the sulphated horizons in the areas of the near-Volga zone of the Sviyaga-Sura divide. It is well known that in this part of the Russian platform the peculiarities in the lithogenesis and palaeogeographical setting of the Ginetian age of the Middle Devonian and of the Lebedyan age of the Upper Devonian, as well as of the Lower Permian time, were favourable for the accumulation of considerable masses of calcium sulphates. It is also known that a certain quantity of calcium sulphates originated and was preserved until now in the lower half of Upper Permian sediments, too.

Certainly, some of these sulphate horizons began to be reduced by bitumens which penetrated there as a result of tectogenesis. It is only in this way that one may explain the large quantity of hydrogen sulphide accumulated in the shape of gas caps in the domes of anticlines, in the region of the Vodoleyev-Sobachkino, Ishterek, Ilyinsk-Vulzyal structures, opened up by the author in 1938, in the course of a study of the «Great Volga» problem. In the region of the town of Cheboksary, a 230 m water column was raised by gases almost to 11 m and is thus far maintained at this height.

As has already been mentioned, in the region of these structures drill-wells have struck at comparatively large depths rocks and waters smelling of petroleum. This facts should be accounted for by the penetration of «dry» and «fat» gases and oil vapours, possibly even with hexane, pentane and other gases. The bulk of either «dry», or «fat» oil gases and vapour was spent on the reduction

of sulphate, but a certain part of the oil vapours and gases, as it must be presumed, continued to migrate, but already with a new companion, *i. e.* with streams of hydrogen sulphide formed as a result of sulphate reduction. Having found their way into the H_2S medium, the oil vapour and gases were, certainly, the best preserved. It is only for this reason that at present the rocks and waters from many deep drill-wells, when opening up Kungurian and even lower Kazan deposits, have a smell of oil.

On the basis of the above data and of assumptions derived from them, the following conclusions can be made:

1. Within the near-Volga zone of the Sviyaga-Sura divide oil-bearing horizons are present.

2. A certain portion of liquid bitumens and their vapour as a result of tectogenesis penetrated into sulphate horizons where they began to reduce the sulphates, with a subsequent formation of considerable masses of hydrogen sulphide.

3. A certain amount of H_2S migrated also from the oil-bearing horizons, but the bulk of hydrogen sulphide originated at the expense of geochemical and biochemical processes of sulphate reduction.

4. In the domes of a number of anticlinal structures there are gas caps, and H_2S issues to the surface along certain fractures in areas comparatively strongly dislocated.

5. The absence of oil shows directly on the surface, or of oxidized remains, is due to the presence of a thick and extensively developed comparatively plastic cover of clays and clay-marlaceous deposits of the Tartar stage. In the southern part of the divide named above this cover consists of Mesozoic deposits, represented mainly by dark and dark-grey compact clays.

The facts here reported, when viewed in the light of our interpretation, confirm the assumption first made by the author in 1931—1932 as to the presence of oil in the near-Volga zone of the Sviyaga-Sura divide ([3]).

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HYDROGEOLOGY

CHEMICAL COMPOSITION OF GROUND WATERS AS DEPENDENT ON SOIL FORMATION

By S. V. ZONN

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The effect of mineralized ground waters upon the salinization of grounds and soils has duly been considered both in soil science and hydrogeology. The reverse relation—changes in the mineral composition of ground waters occurring under the influence of soil formation—have been insufficiently investigated.

The general conceptions as to the development of soils leaves scarcely any doubt as to the rôle of the eluvial process of soil formation in the chemical composition of ground waters which are continuously or periodically in contact with the soil. In literature the author has found only one reference ([5]) to the fact that the degree and partly also the nature of salinity of ground waters may possibly depend on the nature of soil formation above them.

The lack of analytical data on the relation between the soils and the ground waters under the soils made the author select and consider at some length for his work the waters underlying soils of the solonetz type of soil formation, as the best studied ones.

As well known, the formation of solonetz soils is related to the substitution of the stage of salinization by the reverse process, as a result of leaching-out of salts by rainfall and of a drop of the level of ground waters to which these originally owed their salinity.

The analyses of water extracts of soils and of ground waters were recalculated by the author after Palmer, and expressed in per cent equivalents to the composition of both, which excludes any effect of concentration, characterizing only the ratio of the various anions and cations, permitting of a direct comparison with the salt composition of the ground waters and a characteristic of the salinities and alkalinities of soils and waters after Palmer*.

Table 1 contains analyses of the salt composition of soils of solonetz complexes, formed in contact with ground waters, and analyses of ground waters from the trans-Volga region. These analyses show that with an increase in the degree of salinity (analyses Nos. 1, 2) in the soils takes place an accumulation of $Na^{\cdot}$ and a decrease in the content of $Ca^{\cdot\cdot}$, $Mg^{\cdot\cdot}$, Cl' and SO''_4, which are leached out from the soil solutions. The accumulation of $Na^{\cdot}$ is well reflected in Palmer's characteristics, showing the increase in S_1 and A_2, *i. e.* an increase in the content of alkaline cations, connected with strong acids, and in the alkalinity caused by alkaline-earth bases. In ground waters under the solonetz as compared to waters under chestnut solonetz soil, S_2 and A_2 are increased, and primary alkalinity (A_1) appears. The removal of alkalis and their accumulation by ground waters are so considerable as to produce a shift in the Waters from the III to the I class.

The strongly expressed solonetz nature of the chestnut soils (analysis No. 3) in the case of a low level of ground waters shows that in the ground waters, as

* For the soils the average salt composition is given, calculated from the total quantity of anions and cations determined in the whole profile from the surface to the ground waters.

Table 1

Average Content of Water-soluble Compounds in Soils and Ground Waters under these (1–4)

Nos.	Nature of soil and ground waters	Depth of waters, cm	Dry residue	mg-equivalents per cent equivalent to the composition of extract							Characteristics after Palmer				Classes of waters after Palmer
				HCO'_3	CO''_3	Cl'	SO_4''	$Mg^{\cdot\cdot}$	$Ca^{\cdot}$	$Na^{\cdot}+K^{\cdot}$ by difference	S_1	S_2	A_1	A_2	
	Saratov area of the trans-Volga region														
1	Chestnut solonetz	—	0.503	0.48 3.95	0.10 0.82	3.70 30.37	1.81 14.86	—	3.37 27.67	2.72 22.33	44.66	45.80	—	9.54	III
1a	Ground water	410	17.05	2.82 0.65	—	193.72 46.13	12.51 2.92	—	125.02 29.20	89.03 20.80	41.60	51.10	—	1.30	III
2	Solonetz	—	0.606	0.80 14,06	—	0.51 19.26	4.87 16.68	1.31 6.61	2.64 9.95	2.23 33.43	66.80	5.00	—	28.12	III
2a	Ground water	360	1.750	8.57 15.90	0.94 1.74	9.89 18.35	7.55 14.01	2.83 5.24	2.35 4.38	21.77 40.32	64.72	—	16.04	19.24	I
3	Chestnut strongly solonetz soil	—	0.839	0.32 1.29	—	0.65 2.61	11.45 46.10	4.27 17.18	4.04 16.27	4.11 16.5	33.10	64.32	—	2.58	III
3a	Ground water	700—800	2.13	19.56 28.73	—	0.76 1.12	13.73 20.16	2.88 4.23	1,25 1.84	29.90 43.92	42.56	—	45.28	12.14	I
4	Solonetz residual-saliniferous	—	1.08	12.44 20.37	—	0.27 7.79	10.47 21.84	0.86 7.22	1.45 7.28	8.16 35.50	59.26	—	11.74	19.00	I
4a	Ground water	670	0.810	5.16 37.31	—	0.85 6.15	0.90 6.51	2.00 14.49	2.90 21.00	2.01 14.54	25.31	—	3.64	70.98	I
	Near-Caspian lowlands, Djanbek region														
5	Ground water under solonetz slightly leached	569	—	3.44 1.00	—	74.16 21.6	98.74 27.3	24.67 7.2	33.00 9.9	113.67 33.1	66.00	31.80	—	2.2	III
6	Ground water under solonetz medium-leached	512	—	5.08 1.5	—	57.62 17.7	92.80 30.8	10.69 3.3	26.45 8.2	125.36 38.5	77.00	20.00	—	3.0	III
	Southern Ukraine, Genichesk region														
7	Ground water under chestnut soil on slightly saline loess	1325	—	0.388 6.45	—	1340.0 38.94	0.218 4.61	0.01 1.21	0.139 7.03	0.943 41.71	83.43	3.68	—	12.90	III
8	Ground water under solonetz soil in the water	12100	—	0.534 42.6	—	0.038 5.41	0.019 1.99	0.014 5.89	0.066 16.23	0.131 27.88	14.80	—	44.24	40.96	I

compared to soils, S_1 increases, A_2 decreases, and A_1 appears in more considerable amounts. The residual-saliniferous solonetz (analysis No. 4) is characterized by a stage of maximum salinization (both soil and water correspond to the I class). It is of interest to note that Palmer's characteristics show a larger amount of alkalis in soil solutions than in ground water. Most probably the subsequent leaching of salts from the soil will result in a gradual obliteration of the differences in A_2 and an increase of A_1 in the ground water.

The fact that under natural conditions the development of the chemical composition of ground waters follows exactly this course, may be confirmed by the following experimental data. The chestnut solonetz soil under laboratory conditions was washed with water. The analyses of the first and the last portion of the filtrates (Table 2) showed that washing brings about the same effect as that observed in nature. Even a negligible leaching effect was reflected in the analyses which showed an increase in S_1 and A_2 and a decrease in S_2 from the first to the last portion of the filtrate, which is so characteristic of the passage from the chestnut solonetz soils to the solonetz.

Table 2

Variation in Composition of Waters Filtered through 1 m Layer of Chestnut Solonetz Soil (from I. N. Antipov-Karataev ([2]))

Portions of filtrate	Dry residue	Mg-equivalents per cent of total of equivalents in filtrate						Characteristics after Palmer			
		HCO'_3	Cl'	SO_4''	$Mg^{\cdot\cdot}$	$Ca^{\cdot\cdot}$	$Na^{\cdot}+K^{\cdot}$	S_1	S_2	A_1	A_2
Average from first 10 l	9.562	3.4	116.0	24.1	37.6	23.0	82.9	58.8	39.8	—	2.4
		1.2	40.4	8.4	13.1	8.0	28.9				
Same, from 90—100 l	2.276	6.6	19.6	17.7	9.8	8.2	25.9	59.0	26.0	—	15.0
		7.5	22.3	20.2	11.2	9.3	29.5				

The influence of soil formation upon the chemical composition of ground waters can also be observed when comparing the analyses of ground waters taken from soils Nos. 5 and 6 (Table 1). A more intense leaching results in a more intense removal and accumulation of salts which cause an increase in S_1 and A_2.

Analyses Nos. 7 and 8 show (Table 1) the passage of waters under the solonetz in the water «tchay» into the I class, with a prevalence of primary alkalinity (A_1) caused by alkaline bases and weak acids. Under chestnut soils, on the elevations, prevail waters of the III class.

Finally, the same data show that the influence of soil-forming processes upon the chemistry of ground waters may be manifest at a considerable depth of occurrence. According to the classifications of ground waters based on Palmer's characteristics, to the I class belong waters chiefly related to igneous rocks. However, from the example discussed, it follows that the formation of alkaline waters cannot be confined to the influence of alkaline rocks only. They may also be formed under the influence of soil solutions of alkaline type, peculiar of the solonetz series of soil formation.

The rôle of soil-forming processes as of an essential agent in the composition of ground waters has not been given due attention. It seems, however, that each soil type of the eluvial series, such as: podzolized chernozem, chestnut solonetz and other types, by feeding the ground waters with soil solutions of widely different chemical composition, may affect their chemical composition.

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PALAEONTOLOGY

NEW OYSTERS FROM THE PALAEOGENE OF THE TRANS-CASPIAN REGION

By O. S. VIALOV

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The «oyster» collection of the author, kept at the Central Geological Museum in Leningrad, comprises a number of oysters from the Palaeogene of the Trans-Caspian region.

The present paper gives a brief characteristic of some of the most interesting new spesies from the Palaeogene of Mangyshlak (Mangystau) and Tuar-Kyr.

The Mangyshlak oysters—*Gryphaea nomada* sp. n. and *Gr. transcaspia* sp. n.—were found in a series of yellow, greenish and grey Chakyrgan sands and sandstones overlying Danian bryozoan limestones and overlain by nummulitic limestones of the Middle Eocene. These sands contain also the following species: *Liostrea reussi* Netsch., *Gr. sinzowi* Netsch. var. n., *Amphidonta* (*Gryphaeaostrea*) *eversa* Mellev.

With regard to age, the series of sands should probably be referred to the Palaeocene and Lower Eocene (1) and is a stratigraphic analogue of the whole or only of the upper part of the Sogdi division of Central Asia(2) and of the Sumgait of the Caucasus (3).

With regard to *Liostrea labrum* sp. n. it is known only that it comes from the Palaeogene of Tuar-Kyr (well Djaryk), from where the specimens described were furnished by B. A. Fedorovich. True enough, Luppov informed the present author that similar oysters — which are readily identified — has been found by him at Tuar-Kyr (Agoz and Khodja-Su) in the lowermost portion of the Palaeogene section and described in his earlier work by the name of *Dimyodon* (?) sp. (4).

In the author's opinion these beds are analogues of the Chakyrgan sands of Mangyshlak (Palaeocene and Lower Eocene)*.

Family *Ostreidae*
Genus *Liostrea* Douvillé, 1904
Section *Labrostrea* Vialov, 1936

On the inner surface of the lower valve a central sharply depressed portion and lateral flat areas are distinguished. Type—*L. labrum* Vial. (1)

Liostrea (*Labrostea*) *labrum* sp. n. (Fig. 1). Shell small, with an elongated, slightly convex lower valve having an attenuated posterior margin. Apical part of the lower valve small, pointed. The most convex part of the valve is located near the anterior lower margin. Sculpture consists of very fine regular incrementals; in the spaces between these fine concentric grooves are perceivable, in the case of sufficiently good preservation.

* All the figures of the oysters accompanying the present paper have been drawn by V. F. Kurakin.

On the inner surface a central depressed portion and a lateral flat area, particularly wide in the lower and posterior part of the valve, are sharply distinguishable. In the region of development of this flat area the shell becomes markedly thickened. The hinge-area overhanging, flattened. Muscular impression very faint, apparently rounded, located above the middle of the shell and strongly shifted backwards.

Right valve very thin, slightly convex. Hinge area compact, overhanging, bent outwards; its central part is but slightly concave.

Along with the typical specimens (*L. labrum* var. *labrum* var. n.), two varieties, connected with the type of gradual transition, are distinguished.

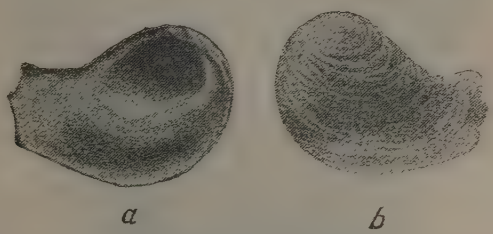


Fig. 1. *Liostrea* (*Labrostrea*) *labrum* sp. n. Lower valve: *a*, internal view; *b*, external view. 1: 1

The first of these—*Liostrea labrum* sp. n. var. *curta* var. n. (Fig. 2) is somewhat higher and shorter. The anterior and the anterior-lower margins of its left valve are more convex. The maximum convexity of the lower margin is located on the line perpendicular to the hinge line, while in *L. labrum* var. *labrum* this maximum convexity of the lower margin is considerably shifted backwards.

In the second variety—*Liostrea labrum* sp. n. var. *plana* var. n. (Fig. 3) the external surface is but slightly convex. Its upper posterior margin is nearly

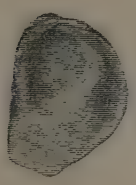


Fig. 2. *Liostrea* (*Labrostrea*) *labrum* sp. n. var. *curta* var. n. Lower valve, internal view. 1 : 1

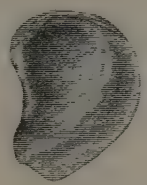


Fig. 3. *Liostrea* (*Labrostrea*) *labrum* sp. n. var. *plana* var. n. Lower valve, internal view. 1 : 1

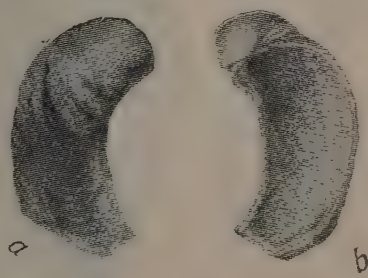


Fig. 4. *Gryphaea* (*Phygraea*) *transcaspia* sp. n. Lower valve: *a*, external view; *b*, internal view. 1 : 1

straight, owing to which the upper part of the flat marginal area of the inner surface is somewhat wider.

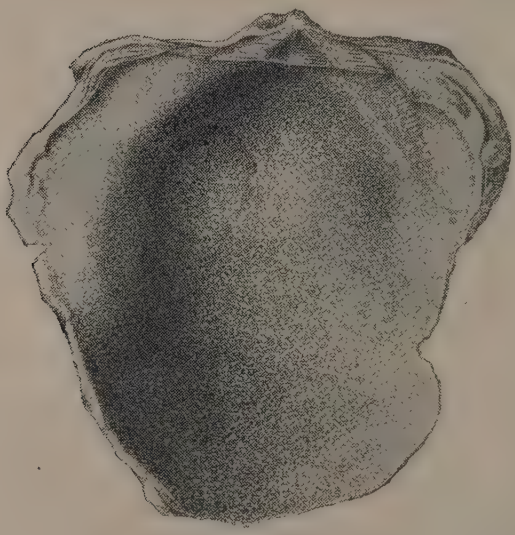
Locality of occurrence: Tuar-Kyr, section 18 at the Djakyr spring. Collection of B. A. Fedorovich.

Genus *Gryphaea* Lamarck, 1801
Section *Phygraea* Vialov, 1936

Apical part slightly convex, shell distinctly asymmetrical, with an attenuated posterior margin. Type: *Gr. frauscheri* Vial. (= *Gr. escheri* Frausch. non M. E.)(1). *Gryphaea* (*Phygraea*) *transcaspia* sp. n. (Fig. 4). Shell small, narrow, crescent, strongly elongated and sharply curved backwards, convex, with a bent apical

a

Fig. 5. *Gryphaea* (*Circogryphaea*) *nomada* sp. n.
Lower valve; external view. $^3/_4$: 1



b

Fig. 6. *Gryphaea* (*Circogryphaea*) *nomada* sp. n.
Lower valve, internal view. $^3/_4$: 1

part, behind which sometimes a small ear-like process is formed. The outer surface is decorated with fine concentric folds, formed by the union of several incrementals.

The inner surface is strongly hollow. The median area is far from reaching the lower posterior margin of the shell. The superficial irregularly rounded muscular impression is hardly perceptible. The comparatively high triangular hinge area is usually somewhat inclined backwards.

The upper valve is not known.

Locality of occurrence: Mangyshlak, Chakyrgan, bed 1. Collection of M. V. Bayarunas.

Distribution: In addition to Mangyshlak, *Gr. transcaspia* sp. n. is known also from the north-western extremity of B. Balkhan (Irtyk-Burun), from where it was determined by the present author in the specimens collected by Luppov ([4]).

Section *Circogryphaea* Vialov, 1936.

Apical part not isolated, hardly projecting, shell more or less rounded, approaching a symmetrical one. Type: *Gr. sinzowi* Netsch. ([1]).

Gryphaea (*Circogryphaea*) *nomada* sp. n. (Figs. 5 and 6). Shell large, with the lower valve of irregularly trapezoid outlines; upper margin long, straight, lower margin narrowed. Valve very strongly swollen (gibbous).

On both sides of the apical portion there are wing-chaped processes; they are usually separated from the dorsal surface by a distinct bend, but occasionally pass into it rather smoothly.

Sculpture consisting of uneven laminated incremental layers. Apical portion non-isolated, hardly projecting over the upper margin.

The median inner cavity, strongly concave, is an oval shape, subdivided into two parts by a gentle ridge stretching from the lower posterior to the upper anterior margin. Muscular impression large, irregularly rounded, at the upper margin markedly hollow. Hinge area not high, triangular. The slightly concave ligament fossula is wider than the quite flat lateral ridges.

Upper valve very uneven, concave, with raised anterior- and posterior-lower margins, covered with sharp non-uniform incremental layers. Ligament fossula markedly hollow.

Locality of occurrence: Mangyshlak, Chakyrgan, bed 2. Collection of M. V. Bayarunas.

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ECOLOGY

ON THE RELATIVELY NARROW RANGE OF DISTRIBUTION OF THE CALCOPHYTES OF THE MIDDLE URALS AS CONNECTED WITH THEIR RELICT NATURE

By A. N. PONOMAREV

(Communicated by V. N. Sukachev, Member of the Academy, 9.XI.1944)

The calcophytic flora of the Middle Urals constitutes a peculiar facies of the local flora which, being of relict origin, has been formed under specific edaphic and historical conditions. It consists mainly of elements of the ancient, tertiary, alpine flora of the Urals, of arctic-alpine species, forms of Siberian light-coniferous forests and of the forest-steppe floral complex. In the composition of this flora an important place is held by species of eastern, in particular, Siberian and eastern-Eurasian distribution; their complex makes up the eastern continental component of the calcophytic flora of the Middle Urals, forming its main core and imparting to it its Siberian character. The eastern component comprises mainly species peculiar to the boreal and mountainous steppes, and inhabiting, besides steppes, also alpine and subalpine tracts. It also comprises coenoelement of the Siberian light-leaved coniferous and small-leaved forests.

The vast majority of the calcophytes of the Ural flora are but relatively stenotopic; they usually have a rather wide ecological distributio being calcophilous along the periphery of their area of distribution only, at exoecic areas and isolated exoclaves alone. Their calcophity is thus a rather conventional and doubtful one, and can by no means be considered as an autochtonic character making part of their ecological constitution. On the contrary, it can be regarded as a new and secondary nature of these species, overlapping and distorting their real original ecological essence.

The calcophytes of the Ural flora are absolutely or relatively relict, degraded forms; their degradation may be a manifest or a latent one, due either to climatic or morphogenetic conditions; they may already be relict forms, or forms approaching the relict condition, being crowded out from some other biotops to limestone soils where they survived as in a refuge, after having lost entirely or partly their phytocoenotic connexions because of a more or less complete degradation of the coenoses they had inhabited before. In the Urals they are very often acoenotic species, representing, as it were, abortive elements of extinct or degrading relict coenoses.

According to their more or less restricted range of distribution, the calcophytes of the Middle Urals can be subdivided into absolute and conventional calcophytes. Taxonomically, these are but stages of calcophyte formation, which express the depth of their relict state and the course of the degradation process of a species, area of distribution and corresponding coenoses.

Absolute calcophytes of a relict type are very numerous among the flora of the Middle Urals. To these belong, for example, *Schivereckia podolica* s. l., *Agropyrum reflexiaristatum*, *Cryptogramma Stelleri*, *Asplenium viride*, *A. ruta muraria*, *Woodsia glabella*, *Dryopteris Robertiana*, *Cortusa altaica*.

Their areas of distribution are of highly fragmented nature; they are absolute relicts, strictly stenotopic everywhere (calcophytic) and acoenotic, for all of them, except *Dryopteris Robertiana*, have lost everywhere the connexion with the coenoses of the recent vegetable cover.

To the conventional calcophytes of the Middle Urals flora belong in the first place some Alpine petrophytic species, as, for example, *Cystopteris fragilis*, *Palypodium vulgare*, *Parietaria micrantha*, *Polystichum lonchitis*, which do not grow on limestone soils only, but on other substrata, as well. These species are an example of a gradual restriction of the edaphic area of distribution and of the destruction of the geographic one, accompanied by a gradual loss of the phytocoenotic relations.

The absolute and conventional calcophytes listed above may, in our opinion, be considered as relict components of an ancient, probably Tertiary, alpine flora of the Urals. It seems rather natural to interpret the calcophily and petrophily of these species as a consequence of their relict condition. This point of view is, however, open to dispute, in so far as the said characters of those species may also be interpreted as connected with their ecological constitution. On the contrary, the idea as to the relict origin and relatively stenotopic nature of the calcophytes finds support in the existence of light-leaved tayga coniferous forest-steppe species of the eastern continental component, and also of the arctic-alpine forms of the calcophytes of the Middle Urals, which we refer to the group of conventional calcophytes.

Especially illustrative from this point of view are those boreal, alpine, steppe-alpine species and light-leaved tayga conifers of the eastern continental component which, being absolutely or mostly calcophytic or sometimes acoenotic at the Middle Urals and in East Europe, have at the same time preserved within the region of the ecological optimum of their main areas of distribution in Siberia a wide ecological range of distribution, and well-pronounced and manysided phytocoenotic relations. Such, for example, are *Adonis sibiricus*, *Allium strictum*, *Allium rubens*, *Alyssum sibiricum*, *Artemisia frigida*, *A. latiflora*, *A. sericea*, *Aster alpinus*, *A. sibiricus*, *Astragalus australis*, *A. depauperatus*, *Aulacospermum multifidum*, *Bupleurum multinerve*, *Centaurea sibirica*, *Chrysanthemum sibiricum*, *Clausia aprica*, *Castilleja pallida*, *Dianthus acicularis*, *D. versicolor*, *Galatella fastigiata*, *Gypsophila altissima*. *Helictotrichon desertorum*, *Larix sibirica*, *Onosma simplicissimum*, *Oxytropis uralensis*, *Polygala sibirica*, *Saussurea propinqua*, *Scorzonera austriaca* ssp. *glabra*, *Seseli Ledebouri*, *Spiraea media*, *Thalictrum foetidum*, *Vicia multicaulis* and other species, having continuous, disjoint or endemic areas of distribution. Among them should probably be classed *Anemone silvestris*, *Carex pediformis*, *Pulsatilla patens*, *Veronica spicata* and others.

In Siberia these species are characterized by a wide oecoareal area of distribution and by their coenotic mode of life. Within a definite range of variation, they grow there in light-coniferous and deciduous forests, upon sandy soil, along the outskirts, in dry birch groves of sparse coppice, in mountainous northern plain steppes, in steppe and occasionally in saline meadows and meadow-steppes, upon grassy hill and mountain slopes and also upon slopes covered with rubbles. Some of these species extend along valleys and mountain slopes of the subalpine and alpine zone. At the periphery, at the ex-oecical occurrences and the exclaves of the western wing of their area of distribution, which is mostly discontinuous, within the limits of the Urals and of the plain of East Europe, these eurytopic Siberian species have a narrower ecological area of distribution; they become either strictly calcophytic, or to some extent so, acoenotic or confined largely to relict coenoses of mountain pine forests and stony steppes. Less frequently they inhabit other coenoses of recent vegatable covering. They are relative relicts and conventional calcophytes. But this is only because of the degradation process which did not affect the whole area of distribution, the species and the coenoses occupied by

the latter, but set in at the western limit of the area of distribution only. The calcophytic and relict condition is not expressed in all species to the same degree. Some of them (*Anemone silvestris, Veronica spicata, Carex pediformis*) may serve as examples of the initial stage of the development of calcophytic mode of life; others may, on the contrary, be considered as future absolute relicts and calcophytes. Such are, for example, *Chrysanthemum sibiricum, Astragalus australis, Aster alpinus, Astragalus depauperatus*, and others.

Only the species of the eastern continental component of the calcophytic flora of the Middle Urals, which have been listed above, are probable relicts of the floristic complex and of the phytocoenoses of the cold, pleistocene pine—larch-birch forest-steppe, as this is understood by I. M. Krasheninnikov.

The calcophytic flora of the Middle Urals and of the Middle Urals forest-steppe, in particular, comprise, moreover, some other steppe species widely distributed in Eurasia anp partly in Eupontic and West Eurasiatic regions as well. These species are *Astragalus falcatus, A. vimineus, A. austriacus*, which can largely be regarded as xerothermic relicts of the steppe stage of the holocene.

Lastly, rather numerous among the calcophytic flora of the Middle Urals are also such arctic-alpine species as *Dryas punctata, Rhodiala rosea, Saxifraga caespitosa, S. cernua, Potentilla nivea, etc.*, which inhabit moist and cold cliffs exposed to the north. These are unquestionable relicts of the glacial epoch and conventional calcophytes, in so far as the latter nature manifests itself in exoecic biotops, arctic and alpine regions, where it is due to their transition to a relict state.

We thus see that the calcophytic flora of the Middle Urals has come into being as a result of a degradation of the landscapes of the pleistocene forest-steppe, of periglacial coenoses of the tundra type, of steppe coenoses of the xerothermic phase of the holocene, by way of accumulation of their relict elements upon limestone soils. Aside from these, it comprises also a number of still older (Tertiary) relicts of the mountainous flora of the Urals, whose landscape and coenotic nature is rather disputable.

In so far as their degradation was largely caused by climatic factors, the calcophytes of the Urals are relicts both morphologically and climatically. These relicts became calcophytes because on limestone rocks they had found a refuge, imitating, owing to the climate prevailing there, the conditions of continental, dry and hot, or cold and humid climate, according to the exposure. These relicts became calcophytes also because their survival upon the limestone rocks was favoured by the fact that they met there scarcely any competition if at all. Consequently, their «calcophytic» habit is a conventional one, being not due to their ecological constitution, but rather to their relict condition and to their survival on limestone rocks that served them as a refuge.

The interpretation of the relict origin of calcophytes as developed above implies a dual nature of their endemism. It assumes that edaphic endemic species, absolute calcophytes in particular, might have arisen not only by way of taxonomic isolation of new forms as a result of adaptation and of an ensuing active morphogenetic process (edaphic neoendemic species of the Cretaceous flora of the South of the European-Russian plain). They might also have arisen by way of a most profound degradation of species, as a result of the reduction of their edaphic and geographical areas of distribution and of the destruction of the coenoses they inhabited.

After these species had lost their coenoses, and their area of distribution had ceased to be continuous, so that they could survive under definite edaphic conditions only, *e. g.* upon limestone substrata, they became palaeoendemic edaphic representatives of the calcophytic flora, and absolute calcophytes of a relict type.

The Molotov State University.

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PLANT PHYSIOLOGY

EFFECT OF GRANULATED SUPERPHOSPHATE UPON YIELD WHEN INTRODUCED WITH SEEDS

By T. T. DEMIDENKO

(Communicated by A. A. Richter, Member of the Academy, 3. VIII. 1944)

The majority of cultivated plants are known to absorb at a young age phosphoric acid which plays an important part in their growth and development, being a constituent of the most complex nucleoproteinic compounds.

Many of crops possessing small seeds strongly react to the phosphoric acid contained in the soil. The reasons are as follows; when the young seedlings appear, they greatly need phosphoric acid, whereas its amount contained in the seeds is a very limited one. The young seedling is at first unable to procure phosphoric acid in great quantities, especially when the soil solution contains a minimum amount of it. When granulated superphosphate is introduced into the soil, phosphoric acid is accumulated locally, and the root hairs are better supplied.

There exist at present two methods of supplying seeds and seedlings with nutrient materials, Gussev's scheme and Avdonin's scheme. In the first method phosphoric acid is glued to the seeds. Avdonin's method is based upon a preliminary saturation of indifferent material (sawdust, husk of rice, millet or buckwheat) with an aqueous extract of fertilizers; when treated with phosphates it forms granules which are dried and mixed with the seeds when sown.

As a medium we have used fresh dry beech sawdust. Since the aqueous extract of superphosphate is usually acid, it is necessary to neutralize it with chalk. Otherwise germination of seeds is prevented by the excess of acidity and the single seedlings formed are rapidly killed.

The experiments were conducted at the Zakataly Experiment Station in the Azerbaidjan SSR.

For brevity sake, we shall present here the results relating only to two periods, *viz.*, flowering and full maturity (Table 1).

It may be seen from the data of Table 1 that in the case of millet and sesame there was a considerable effect of the phosphoric acid introduced at the time of flowering. The highest rate of growth was shown by the plants which received 10 kg P_2O_5 per hectare into the rows; the second place have assumed plants which received 45 kg P_2O_5 per hectare in the form of granulated superphosphate; next followed plants which were given 45 kg P_2O_5 per hectare; the fourth place was assumed by plants which received 5 kg P_2O_5 per hectare in the granulated form into the rows at the time of sowing. The last place belonged to plants which received 15 kg P_2O_5 per hectare either in the form of ordinary, or granulated superphosphate.

An analysis of the biometric indices derived from all the experiments has shown that the start given to the seedlings under the influence of P_2O_5 was preserved during the whole growing season. This should be explained by the fact that phosphoric acid kept on acting positively upon the growth of the plants until maturity.

In concluding the following should be noted: the phosphoric acid which gives a different start to the young plants continues acting until the plants attain full maturity. Maturation set in at one, since the plants absorbed a sufficient amount of phosphoric acid which enhanced maturation of millet, sesame and oil flax.

Table 1

Effect of Granulated Superphosphate upon Development of Millet and Sesame

Experimental scheme	Height of the plants in cm			
	Millet		Sesame	
	at the time of flowering	at full maturity	at the time of flowering	at full maturity
No fertilizers	39.2	50.3	35.4	46.4
5 kg P_2O_5 per hectare introduced into the rows in the form of granulated superphosphate	44.4	52.2	37.8	49.5
10 kg P_2O_5 per hectare introduced into the rows in the form of granulated superphosphate	49.4	59.5	39.5	51.2
15 kg P_2O_5 per hectare introduced into the rows in the form granulated superphosphate	48.5	58.5	38.6	50.5
15 kg P_2O_5 scattered in the form of ordinary superphosphate	42.4	52.4	36.5	49.6
15 kg P_2O_5 scattered in the form of granulated superphosphate	43.6	53.4	38.4	50.5
45 kg P_2O_5 scattered in the form of ordinary superphosphate	46.5	57.5	39.4	51.3
45 kg P_2O_5 scattered in the form of granulated superphosphate	48.2	58.5	40.2	52.4

In the course of the growing period nitrates, phosphates and pH were determined under the plants. Observations on the dynamics of the nitrates have shown that the latter were present under the plants until flowering set in, after which they began to disappear.

In all plots, at any date the amounts of NO_3 proved to be rather similar, which points to the fact that phosphoric acid has but little effect upon the processes of nitrification.

Owing to cultured condition and high fertility of the soil it proved possible to create favourable conditions for the accumulation of nitrates, so that the plants received a considerable amount of nitrates at an early age. As to the dynamics of the phosphoric acid in the soil which was differently fertilized, the greatest amount of it was discovered at any date in plots which had received 15 kg P_2O_5 per hectare into the rows together with the seeds. Much phosphoric acid was also found in plots which were given 45 kg P_2O_5 per hectare by scattering.

A much smaller amount of phosphoric acid was found in plots which have received 5 and 10 kg of phosphoric acid per hectare by introducing it into the rows. The last place as to the amount of phosphorus was assumed by the plots which have received 15 kg P_2O_5 scattered per hectare in the form of ordinary or granulated superphosphate.

The amount of water-soluble P_2O_5 was found to decrease as it was consumed by the plants.

The active acidity (pH) was but slightly altered under the plants in the course of the growing season, owing to high buffer properties of the soil.

Maturation of the plants proceeded under rather severe conditions: the weather was dry and hot, and the distribution of the plastic substances was not quite satisfactory. The data on the yield are presented in Table 2.

It may be observed from the below data that all the three crops react positively to the introduction of phosphoric acid. The positive reaction of these plants to phosphorus depends on the fact that this element is present in this particular soil in an insufficient amount.

Table 2

Effect upon the Yield of Ordinary and Granulated Superphosphate Introduced when Sowing Seeds

Experimental scheme	Yield of millet		Yield of sesame		Yield of flax	
	c per ha	per cent	c per ha	per cent	c per ha	per cent
No fertilizers	7.05	100	3.30	100	8.14	100
5 kg P_2O_5 per hectare in the form of granulated superphosphate introduced into the rows	7.87	111.6	3.76	113.9	8.94	109.8
10 kg P_2O_5 per hectare in the form of granulated superphosphate introduced into the rows	8.70	123.4	3.93	119.0	9.63	118.2
15 kg P_2O_5 per hectare in the form of granulated superphosphate introduced into the rows	8.77	124.4	3.95	119.7	9.75	119.7
15 kg P_2O_5 per hectare in the form of the ordinary superphosphate scattered	7.67	108.8	3.58	108.5	—	—
15 kg P_2O_5 per hectare in the form of granulated superphosphate scattered	7.82	110.9	3.67	111.2	—	—
45 kg P_2O_5 per hectare in the form of ordinary superphosphate	8.42	119.4	3.80	115.2	9.43	115.9
45 kg P_2O_5 per hectare in the form of granulated superphosphate	8.6	121.9	3.96	120.0	9.92	121.8

Flax reacted positively to the introduction of ordinary and granulated superphosphate, and so did millet and sesame.

As to the effect of the introduction of the granulated superphosphate into the rows, it should be noted that under the meteorological conditions of the given summer a good effect was observed with 10 kg P_2O_5 per hectare on the soil in question. Introduction of 15 kg per hectare is followed by a similar effect, which might be accounted for by the dry summer resulting in an increased osmotic pressure of the soil solution, causing a suppression of plants. 5 kg per hectare resulted in a lesser increase than 10 kg per hectare.

Scattering of 15 kg P_2O_5 in the form of the ordinary or granulated superphosphate produced less effect than 5 kg of granulated one introduced into the rows.

Introduction of 45 kg P_2O_5 per hectare produced a considerable effect upon the yield of millet, sesame and oil flax, the effect being somewhat greater in the case of granulated superphosphate than in the case of the ordinary one.

The experiments permit of the following conclusions:

1. Granulated superphosphate introduced into the rows has a positive effect upon the growth and development of plants.

2. The physiological start received by the plants at an early age and resulting in different rates of growth and development is preserved during the whole period of growth, inclusive of the time of flowering and maturation.

3. The greatest amount of phosphoric acid was found under the plants in the rows where 15 kg P_2O_5 per hectare was given; the second place was assumed by the dose of 10 kg; the third, by the dosage of 5 kg per hectare.

4. When 45 kg P_2O_5 per hectare was scattered, less phosphoric acid was found in the plots than when it was introduced into the rows.

5. The most effective and economically profitable method proved the introduction into rows with seeds 10 kg P_2O_5 per hectare.

6. The coefficient of assimilation of phosphoric acid sharply increases when it is introduced with seeds into rows in the form of granulated superphosphate.

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PLANT PHYSIOLOGY

ON THE EFFECT OF HETEROAUXINE UPON CELLS OF DISEASED PLANT

By K. SUKHORUKOV and B. STROGONOV

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There exist different hypotheses as to the mechanism of the influence of the heteroauxine and of substances similar to it upon the plant cell. Auxines are supposed to increase the plastic tensility of the external membrane of the cell, to rise osmotic pressure, act directly upon the cytoplasm, modify the cell's ability to absorb water and solutes ([1–3]). Eyster ([7]) has suggested that the auxines act upon the cell by inhibiting or activizing the enzymes in the presence of adsorbents. These hypotheses do not exclude one another and are theoretically applicable to the cells of higher and lower plants, although the physiological effect of the auxines on the two groups of plants is known to differ greatly.

The formation and accumulation of the auxines within the cells of both normal and diseased plants is usually accompanied by the accumulation of the cell division hormones or of the substances of bios. The bios substances are present in the plant cells either in combination with the cell proteins or in a free state ([1, 6]). The bios substances and the proteins may become connected by strong bounds which are broken when the breakdown of the protein takes place. The method of separating bios developed by Nielsen and Hartelius in 1938 ([9]) is based on the ability of the bios to be absorbed by the cells when its solution is shaken with yeast. The bios substances are of great importance for the cell's life, especially at the period of its division and growth. Like the majority of heterotrophic organisms parasitic organisms need for their development ready bios substances, the hormone factor.

Auxines of the type of heteroauxine are formed by fungi and bacteria, but their physiological rôle in these organisms remains obscure; their origin is connected with nitrogen exchange, with the form of nitrogenous nutrition, with the transformation of certain aminoacids. Abnormalities in the growth of plants caused by patogenic fungi and bacteria are now related to the auxines formed by the parasite. The very fact can scarcely be questioned, but the physiological rôle of such excretes and of their effects remains rather obscure. It was our suggestion that, by excreting auxines, the parasites modify the distribution of other hormones which are important for their nutrition and growth. Cooper arrived at the same hypothesis in his studies of the effect of heteroauxine upon root formation ([5]). He believes indolyl-acetic acid to cause shifts of other substances within the plant, these substances being necessary for the root formation.

To check up our hypothesis we have arranged a number of experiments. Lamellae were prepared from the cortex of the carrot root, the dimensions being $45 \times 15 \times \times 2$ mm. After being rinsed in water, the lamellae were immersed (to the half of their length) into solutions of synthetic indolyl-acetic acid, the concentration

being 0.01, 0.1 and 1.0 per cent. The heteroauxine was dissolved in alcohol and the solution diluted with water (the ratio of alcohol to water was 5 to 95 volumes). After 24 hours the lamellae were taken out, rinsed with water, cut into two parts (the lower immersed and the upper which remained outside the surface of the heteroauxine solution), the halves were ground with the aid of a mortar, a decoct was prepared from the resulting pulp; the decoct was filtered, several times extracted with pure ether, transferred to a metric flask and diluted with water to a ratio of the weighed portion: water equal 1 : 2. The decocts were tested for the bios content by adding 1 cm^3 per 20 cm^3 of the Gaiduk's medium inoculated with homogeneous suspension of yeast (*Saccharomyces cerevisiae*, Saratov strain, obtained from Dr. A. R. Verner). Inoculated flasks were left at 18° C for 80 hours, and the growth of the yeast was arrested by placing them into a refrigerator. The dry weight of the yeast was recorded after the content of the flasks was filtered through crucibles with glass filters of suitable density. The results of experiments 1 and 2 are presented in Table 1; those of the 3rd experiment are contained in Table 2.

Table 1

Content of Bios in Carrot after Treatment with Heteroauxine (dry weight of the yeast in mgs)

Concentration of heteroauxine	Lower part	Upper part	$\frac{\text{Upper}}{\text{Lower}} \times 100$
I experiment			
Control (95 cm^3 of water, 5 cm^3 of 95% alcohol)	24	24.4	101.6
0.01% . .	21.5	33.5	155.8
0.1% . . .	4.5	27.4	609.0
1.0% . . .	6.5	22.5	346.1
II experiment			
Control (95 cm^3 water, 5 cm^3 of 95% alcohol) .	38.4	43.7	113.8
0.01% . . .	40.0	47.4	118.5
0.1% . . .	10.8	32.0	295.4
1.0% . . .	16.3	45.4	278.5

Table 2

Content of Bios in Carrot after the Extracts were Treated with Activated Carbon; the Bios has Penetrated into the Liquid by Diffusion (dry weight of the yeast in mgs)

Material studied	Lower part	Upper part	$\frac{\text{Upper}}{\text{Lower}} \times 100$
III experiment			
No treatment with activated carbon:			
Carrot, control$_3$ (95 cm^3 of water, 5 cm^3 of 95% alcohol)	21.5	28.4	132.1
Carrot in 0.1% of heteroauxine	16.4	33.8	206.1
Treated with activated carbon			
Carrot, control	0.4	6.2	1550
Carrot, in 0.1% of heteroauxine	0.8	2.5	312.5
Diffusion of bios from the carrot			
In 95 cm^3 of water plus 5 cm^3 of 95% alcohol .	2.4	—	—
In 0.1% of heteroauxine	3.8	—	—

In Table 2 are presented the results of the investigation of the extracts for the bios content before and after treatment with carbon (30 g per 100 cm³ of extract, stirring for 15 min., filtering and washing with water followed by bringing the volume up to the initial value by dilutions with water). In the same table are contained data on the diffusion of the bios from the experimental sections of carrot into the liquids applied. The quantity of the liquid introduced into the medium for the yeast corresponded to a weighed portion of carrot amounting to 0.3 g, or to 1 cm³ of the decoct.

As may be seen from the figures presented, heteroauxine modifies the distribution of bios within the tissues. It is possible that this effect is connected with the poisonous properties of heteroauxine, and with a modification of the colloids, since a noticeable shift of the bios was observed under the influence of a pure 5 per cent solution of alcohol (from 1.6 to 32.1 per cent). Bios is able to diffuse from the cells, and heteroauxine enhances diffusion, probably, by increasing the permeability of the cytoplasm of living cells. It was observed that substances both easily and sparingly absorbed by activated carbon were shifted (Table 2), *i. e.* independent ly of their surface activity.

By the term «bios» we do not understand any definite individual chemical component, but rather a complex of substances which are necessary for the heterotrophic nutrition of the cell, in the form of supplementary nutritive material. In the recent years when delicate chemical methods made it possible to identify a considerable fraction of the substances belonging to the bios group, the term «bios» gradually vanishes from the pages of biological literature, its importance being a rather historical one. We understand that «bios» in the above interpretation has a definite physiological meaning, and that changes in its quantity make it possible to judge of the state of the cell and of its relation to the surroundings. Within the last decade many physiological studies on bios were published in the USSR (on the hormones of cell division), but a considerable part of the Russian papers have been overlooked by the Western workers, which may now be seen from the newly formulated laws and ideas ([3]) known in this country long since.

The effect of heteroauxine upon the movement of other substances was not studied by us in detail; a few determinations relating to nitrogenous substances are presented below. Total nitrogen (after Kjeldahl) was determined in the cortex of the carrot treated after the above-described method with 0.1 per cent heteroauxine solution. Nitrogen content was calculated in mgs per 100 g of raw material.

	Lower part	Upper part
In 0.1 per cent heteroauxine solution	82	79
Control (water plus alcohol) . . .	80	82

No changes in nitrogenous substances content were observed. Nitrogeneous substances are also contained in the bios group, but, naturally enough, the nitrogen contained in them is so scarce that it cannot tell upon the total content.

Heteroauxine may thus cause a redistribution of the cell-division hormones in the plant tissues. It may be suggested that the parasites modify the distribution of cell-division hormones in the diseased plant by liberating auxines and thus create for them favourable conditions of nutrition and growth.

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PHYTOPATHOLOGY

MOSAIC ON *LEGUMINOSAE*

By A. M. VOVK

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Virus decreases are widely distributed among leguminous plants in Europe, America, Asia and Australia; they do much harm to *Phaseolus*, soy-bean and other species of this order ([1;5. 6, 11–19, 21, 22]). Most of virus diseases affecting *Leguminosae* are transmitted by various species of aphids, or by way of inoculation with sap. Some of them are also transmitted through infected seeds, as, for example, *Phaseolus* virus 1 Pierce and the soy-bean mosaic. A number of viruses cannot be transmitted otherwise than by transplantation ([1, 2, 5–7, 13, 17, 21, 22]). The taxonomic aspect of the virus diseases of *Leguminosae* has not been investigated to a sufficient extent ([5–7, 15, 17, 21, 22]). In the USSR virus diseases have been reported for *Phaseolus*, soybean (pea), ground-nut and *Crotalaria* ([1–5, 8, 9]). Rischkov discovered at the Ufa province mosaic on alfalfa in 1941 (unpublished data).

Dunin, Yakimovich and Kozlova have demonstrated that *Phaseolus* and soybean are especially liable to be attacked by virus diseases. According to the abovenamed writers, these crops show occasionally a 100 per cent infection with mosaic ([1, 2, 4, 9, 10]). Despite the high economical importance of the virus diseases affecting leguminous plants, only the mosaic on soy-bean has been studied in some detail. Yakimovich has found that the mosaic on soy-bean is transmitted by various species of aphids, by sap inoculation, and also through seeds ([9, 10]).

As to the rest of virus diseases discovered on leguminous plants, they have not been studied at all. Entirely lacking is information on the nature of the various viruses, and on the correlation of the diseases caused by them with definite taxonomic units. The regularities underlying the spread of these diseases are still obscure. The mosaic on *Phaseolus*, which was described by Iwanowski as early as in 1902, has still remained beyond the scope of investigations ([3]).

On a plot of the All-Union Agricultural Exhibition we discovered in 1940 a wide spread of mosaic by which were badly affected *Phaseolus*, soy-bean, sowing and sweet-pea, and *Vicia faba*. The disease was manifest in the form of light-green mosaic and crimpage of the leaves. The affected leaves were often chlorotic, covered with scattered spots. The disease resulted in an inhibited growth of the plants and in a reduced size of the fruit. In the case of mosaic on soy-bean we observed symptoms similar to those described by Yakimovich. In pea the disease was expressed in the form of light areas along the venation of leaves, in a slight deformation, and also in an insignificant inhibition of growth.

Similar phenomena could also be observed in sweet-pea. In beans (*V. faba*) the disease took the shape of a slight marmoreal mosaic of the leaves. The plants affected by the mosaic showed occasionally inhibited growth or dwarfish leaves. In the environs of the town Frunze (the Kirghizian SSR) we observed in 1943 *Phaseolus*, soy-bean, alfalfa and clover fields affected by mosaic.

Our experiments were undertaken with the aim to find out whether it is possible to transmit the mosaic from bean, pea and sweet-pea to some other species of *Leguminosae* through aphids, or by way of inoculation with sap.

For the experiments with aphids use was made of sex-mature and larval forms of *Myzodes persicae*; the experiments were carried on in a greenhouse, under conditions of isolation. The Aphids were taken from a healthy tobacco plant and transferred to bean and pea-plants affected with mosaic. After 5 to 17 hours, the aphids were transferred to unaffected seedlings of pea, bean or *Phaseolus*. The latter procedure was performed with the aid of glass-tubes with their ends open, which were put on the leaflets of the plants and stopped with cotton. To each plant was transferred one tube containing 10—20 aphids. For the sake of control part of the aphids were transferred to healthy plants, without being preliminarily fed on plants affected by mosaic. There were also such control plants of the three species, to which aphids were not transferred at all. In this way we succeeded in transmitting the infection. The symptoms of the disease became manifest after some 11—17 days. The bean-mosaic was thus transmitted to bean, pea and *Phaseolus* plants. In bean-plants the disease took the form of a slight, light-green mosaic of the leaves, while in pea and *Phaseolus* it produced light-green areas along the leaf venation, and the appearance of green and yellow spots.

The virus of the seed and sweet-pea could readily be transmitted through aphids to bean, pea and *Phaseolus*. In the two former species the disease at its outset was accompanied by the appearance of a light area along the leaf venation; latter on, green and yellow spots appeared on the leaves. In *Phaseolus* it was manifest in a light-green mosaic. The control plants remained sound in all the three species (Table 1).

Table 1

Experiments on the Infection of Various Leguminosae

Source of infection	Crop	Variety	Number of plants	
			under experiment	diseased
Mosaic bean	Bean	Chernorussky	11	5
	Pea	Rostovsky	10	2
	Phaseolus	Wilhelm	10	7
Mosaic sowing-pea . . .	Bean	Chernorussky	10	9
	Pea	Rostovsky	10	4
	Phaseolus	Wilhelm	10	2
Mosaic sweet-pea . . .	Bean	Chernorussky	8	2
	Pea	Rostovsky	13	4
—	Bean (control)	Chernorussky	19	—
—	Pea (control)	Rostovsky	40	—
—	*Phaseolus* (control)	Wilhelm	10	—

By inoculating the plants with sap it was found possible to transmit the bean-mosaic to bean and pea, while the attempts to transmit it to *Phaseolus* proved a failure. Bean, pea and *Phaseolus* plants were infected with the sap of pea-plants affected by mosaic, and the mosaic of sweet-pea was transmitted to seed-pea. The symptoms of the disease in bean and pea-plants took the form of light areas along the leaf venation and of light-green mosaic; in *Phaseolus* it appeared in the form of light-green mosaic only. The disease became manifest some 12—16 days upon inoculation. The plants affected with the mosaic were usually somewhat lagging behind in their development (Table 2).

The mosaic on seed-pea, sweet-pea and bean seems to be caused by the *Pisum* virus 2 Doolittle and Jones (according to Smith's nomenclature); like the latter, it is readily transmitted both by aphids and sap, affecting pea, *Phaseolus* and bean. Its symptoms are similar to those of the *P.* virus 2.

Table 2

Mosaic of *Leguminosa* as Transmitted Through Inoculation with Sap

Source of infection	Crop	Variety	Number of plants	
			under experiment	diseased
Sap of mosaic bean	Bean	Horse bean	9	5
	Pea	Rostovsky	9	6
	Phaseolus	Triumph	9	—
Sap of mosaic sowing-pea	Bean	Horse bean	8	5
	Pea	Rostovsky	20	2
	Phaseolus	Triumph	10	7
Sap of mosaic sweet-pea	Pea	Rostovsky	12	2
	Bean (control)	Horse bean	10	—
	Pea (control)	Rostovsky	10	—
	Phaseolus (control)	Triumph	20	—

As well-known, *Cucumis* virus 1 Doolittle is capable of affecting various species of *Leguminosae* ([7, 17, 20]). In order to clear up the question, whether the mosaic discovered on leguminous plants is or is not caused by the *Cucumis* virus, experiments were carried out, involving the transmission of bean and pea mosaic to cucumber seedlings; the infection was effected through aphids and by sap inoculation. Our experiments have shown that the mosaic of bean and pea produces no morbid phenomena in cucumbers, which should be taken to mean that this virus is not identical with the *Cucumis* virus. The attempts to transmit with the aid of aphids or sap inoculation the bean and pea mosaic to *Nicotia tabacum* and *N. glutinosa* also proved unsuccessful.

In order to control the spread of virus diseases affecting leguminous grain-crops it is necessary to exterminate by all possible means the aphids which are the transmitters of the mosaic of *Leguminosae*. Along with measures for controlling the pest, good results are also likely to be obtained by breeding varieties resistant to the mosaic of *Leguminosae*. A number of pea varieties have been found by Chamberlain ([12]) to be immune against the mosaic.

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EXPERIMENTAL MORPHOLOGY

CHEMICAL METHODS FOR RESTORING THE REGENERATIVE CAPACITY OF LIMBS IN TADPOLES

By L. W. POLEŽAJEW

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In our earlier experiments we succeeded in restoring or prolonging the regenerative capacity of the limbs in tadpoles of *Anura* by using different methods: traumatization ([2, 3, 5]), transplantation ([1, 7]), re-regeneration ([2, 3]), repeated amputation ([3]). In all cases an intense dedifferentiation of the tissues of mesodermal origin took place in the stump, as well as the liberation and accumulation of cellular material on the amputation surface, and the formation of a regeneration blastema. In 1942 appeared an interesting article by Rose ([9]) who had succeeded in restoring the regenerative capacity of limbs in adult frogs with the aid of a chemical method, *viz.* by treating the stumps with a saturated solution of sodium chloride. In his brief note this writer gives no description of the alterations that took place in the tissues as a result of the salt treatment. The interest aroused by Rose's new method made me apply it to experiments with adult frogs and tadpoles. The results of the experiments with frogs will be reported elsewhere; the present article is concerned with the results of experiments on tadpoles. The latter were chosen as material on account of the high rate at which regeneration proceeds in them; besides, the results obtained with them could be compared with our previous data secured with the aid of the methods earlier described. In this way it appeared possible comparatively to evaluate the efficiency of the new method and the old ones.

The efficiency of Rose's method might be due to two different circumstances. First, it could be due to the specific nature of the chemical treatment; secondly, to the long period of the treatment which lasted up to two months, whereas in our previous experiments involving traumatization, transplantation and other treatments, all of them were but of short duration and were never repeated. In order to check up the specific effect of the chemical substances applied by Rose, various chemical agents were put to test, *viz.* NaCl, glucose, iodine and acid; the results obtained were compared with the data secured by means of traumatization and transplantation. In order to check up the second circumstance, experiments were undertaken involving prolonged traumatization at the stage IIb, in four-fold replication.

Two modifications of the treatment with chemical substances were applied: 1) individual treatment and 2) total treatment. In the former case, the tadpoles had the two hind limbs cut off; the stumps of the left ones were subjected to no subsequent treatment and served as control, while those of the right ones were during 1—4 minutes exposed to the effect of the chemical agent under test. The solution was imbibed in a glass capillary, the end of which was applied to the wound surface. In the experiments with iodine, the latter was

applied to the wound surface in the form of solution. Individual treatment was applied 1—2 times during 24-hour-day, no more than 4—5 times during the experiment. In the case of total treatment, the animals had only their right hind limb amputated. The control animals were placed in water, the experimental ones, in a solution of the chemical substance to be tested, for a period up to 260 hours. In the course of the experiment, however, the animals were alternately kept in solution and in water. Their uninterrupted stay in the solution varied from 7 to 24 hours.

The following chemical agents were put to test:

Sodium chloride, saturated solution in the case of individual treatment, and 0.5 per cent solution (tolerable concentration) in the case of total treatment;

iodine, 10 per cent alcohol solution, for individual treatment;

acid (HNO_3), 10 per cent water solution of fuming nitric acid, for individual treatment;

glucose, 5 per cent solution for total treatment;

colchicine, 1 : 1000 solution.

Tadpoles of *Rana temporaria* were taken at the I stage, when their hind limbs still conserve their regenerative capacity, and at the stages IIa, IIb and IIIa, when they have already lost this capacity ([4]). On account of the importance attached to the level of amputation ([4,10]), the limbs were cut off at the proximal end of the shank.

When individual treatment was applied, as positive results were considered such cases where the development of experimental limbs was found to prevail over that of the controls; the opposite case was considered as a negative result. With total treatment as positive results were considered such cases where there was regeneration in the experiment, and as negative, cases showing regeneration in the control. In the two kinds of experiment, as total positive results were considered such cases as represented the difference between the positive and the negative results. The data of microscopic examination will be the subject of a special report. The results of macroscopic investigation are given in Tables 1 and 2.

In the case of restoration of the regenerative capacity at the IIa stage, the total positive result of regeneration in our earlier experiments with single traumatization attained 52.1 per cent ([5]); in our experiments with transplantation it was 60.0 per cent([7]); in the experiments here described it was in the case of individual treatment with salt, 42.33 per cent; with iodine, 31.46 per cent; with nitric acid, 39.9 per cent; in the experiments involving total treatment with glucose the positive results attained 45.0 per cent; and with salt, 9.3 per cent only. These date demonstrate that among the chemical agents put to test the most efficient is the glucose. However, its effect did not exceed that of traumatization or transplantation, but was even found to be somewhat below it. From our experiments involving treatment with salt it is evident that the efficiency of this treatment depends on the concentration of the solution: thus, the effect produced by a saturated solution is much stronger than that of a dilute one (0.5 per cent).

When the treatment was applied at the IIb stage, the total positive result of regeneration in experiments involving single traumatization([5]) was found to make up 17.1 per cent, partial regeneration comprised; in the experiments with glucose it was 50.1 per cent, whereas in those with iodine and total salt-treatment, there was no regeneration at all.

If the treatment was applied to animals at the IIIa stage, which gave no regeneration in the control, the total positive result with glucose attained 30.0 per cent, but at the expense of partial regeneration only.

The above data, obtained when experimenting at the IIb and IIIa stages, may lead one to conclude that glucose is a specifically efficacious agent, whose effect is to stimulate regeneration in such cases where any other agent proves a failure, or is but poorly efficient. Such a conclusion would be premature, however. First, the number of chemical substances applied to animals at this stage was rather small: secondly, experiments involving repeated traumatization went to demonstrate that other methods may be no less efficient if applied at these stages.

Experimental set	Stage when operated	Total number of animals operated upon	Survived and recorded	Fixed at early stages		Complete regeneration of both limbs		Partial regeneration of both limbs		Neither of the two limbs regenerated		Posi	
				No definite result	Positive result	Number of cases	Per cent	Number of cases	Per cent	Number of cases	Per cent	Complete regeneration of the right limb; the left one failed to regenerate	
												Number of cases	Per cent
Repeated traumatization	IIb	72	48	—	8	—	—	5	1.04	11	22.92	10	20.83
Individual treatment with salt	IIa	129	85	7	3	17	20.0	6	7.06	20	23.53	23	27.06
Individual treatment with iodine	IIa	167	89	24	14	6	6.74	5	56.18	40	44.94	11	12.36
Do	IIb	21	18	3	—	—	—	—	—	18	100.0	—	—
Individual treatment with nitric acid	IIa	30	27	—	1	2	7.41	2	7.41	10	37.03	3	11.11

The total positive result of regeneration in the case of treatments applied at the IIb stage in experiments with single traumatization ([5]) makes up 17.1 per cent. However, if traumatization is practised during 4 days in four-fold replication, the respective data is 54.17, and in the case of glucose treatment applied during

Table 2

Experimental set	Stage of operation	Total number of operated animals	Survived and recorded	Fixed at early stages		Complete regeneration		Partial regeneration		No regeneration		Positive result		Total positive result, per cent
				No definite result	Positive result	Number of cases	Per cent	Number of cases	Per cent	Number of cases	Per cent	Number of cases	Per cent	
Control (simple amputation)	I	30	21	9	—	17	80.05	2	9.52	2	9.52	19	89.6	—
	IIa	25	25	—	—	9	36.0	—	—	16	64.0	9	36.0	—
	IIb	110	90	—	—	—	—	21	23.3	69	76.7	21	23.3	—
Colchicine	I	67	44	11	—	39	88.6	1	2.3	4	9.1	40	90.0	0
Glucose (total treatment)	IIa	43	31	1	7	19	61.3	3	9.7	9	29.0	22	71.0	45.0
	IIb	25	15	6	4	7	46.7	4	26.6	4	26.6	11	73.4	50.1
	IIIa	30	10	11	6	—	—	3	30.0	7	70.0	3	30.0	30.0
Salt (total treatment)	IIa	57	53	—	4	19	35.9	5	9.43	29	54.7	24	45.3	9.3
	IIb	57	40	5	2	5	12.5	1	2.5	34	85.0	6	15.0	0
	IIIa	23	15	7	1	—	—	1	6.7	14	93.3	1	6.7	—

Table 1

tive result						Negative result						Total positive result, per cent
Partial regeneration of the right limb, the left one failed to regenerate		Complete regeneration of the right limb, partial regeneration of the left limb		Summarized		No regeneration of the right limb, partial regeneration of the left one		Partial regeneration of the right limb, complete regeneration of the left one		Summarized		
Number of cases	Per cent	Number of cases	Per cent	Number of cases	Per cent	Number of cases	Per cent	Number of cases	Per cent	Number of cases	Per cent	
11	22.92	8	16.67	29	60.42	3	6.25	—	—	3	6.25	54.17
8	9.41	8	9.41	39	45.86	2	2.35	1	1.18	3	3.53	42.33
18	20.22	4	4.49	33	37.08	5	5.62	—	—	5	5.62	31.46
—	—	—	—	—	—	—	—	—	—	—	—	—
4	14.81	4	14.81	11	47.4	1	3.7	1	3.7	2	7.41	39.9

115 hours in the course of 8 days it is 50.1 per cent. It can thus be assumed that it is rather the duration of an intense treatment than its nature (traumatization, application of glucose, salt or of other chemicals) that plays a decisive part in the restoration of the regenerative capacity.

The analysis of the data on the effect of colchicine upon the regenerative capacity are partly beyond the scope of the present article. The problem we set before us was to clear up whether it is possible to restore the regenerative capacity lost by an amputated limb of a tadpole by acting upon its stump with various chemicals. The experiments with colchicine were undertaken in order to suppress the regeneration of limbs in tadpoles at such stages (stage I) when no regeneration should take place under normal conditions. Thornton ([11]) has shown that the effect of colchicine applied in a 1 : 1000 and 1 : 1500 dilution is fully to inhibit the regeneration of limbs in *Amblystoma* larvae. In our experiments, however, colchicine applied in a 1 : 1000 dilution during 260 hours in the course of 19 days failed to suppress the regeneration of limbs in tadpoles of *Rana temporaria* amputated at the I stage. Nothing except an insignificant lagging in the development of regenerates could be observed. The disagreement between the results obtained by Thornton and our results should evidently be put down to the fact that different material was used in the two cases. Further experiments have to demonstrate whether the difference in the behaviour of the tissues of regenerating limbs of frog tadpoles and larvae of *Amblystoma* depends upon the degree of their resistance to colchicine, or is consequent to a different trend in the regeneration processes, inherent in each of the two species. From our earlier data ([8]) it follows that in the case of limb regeneration in tadpoles the mitotic coefficient is considerably lower, while the phenomena of tissue dedifferentiation of the stump are more pronounced than in the case of limb regeneration in axolotls. If the process of cell division alone were affected by colchicine, then the divergence between the

results of the experiments here described and Thornton's data ([11]) would be quite comprehensible. However, the effect of colchicine is to destroy the structure of the amputated limb, too. It seems, therefore, plausible to assume that the tissues of the limbs of tadpoles are more resistant to the effect of colchicine than the tissues of *Amblystoma* larvae. It is possible that higher concentrations of colchicine will produce an inhibitive effect upon the regeneration of limbs in tadpoles.

Conclusions. 1. In tadpoles of *Rana temporaria* the lost regenerative capacity of the hind limbs can be restored with the aid of chemical methods: by individual treatment of the wound surface with a saturated solution of sodium chloride, 10 per cent alcohol solution of iodine, 10 per cent solution of nitric acid, or by a total treatment of the animal with a 5 per cent solution of glucose.

2. Under the conditions of the experiment here reported glucose proved to be the most efficient agent.

3. The chemical methods are not more efficacious than the methods of traumatization or transplantation, by using which it is possible to restore the regenerative capacity of limbs in tadpoles.

4. The chances to restore the regenerative capacity of limbs in *Anura* depend upon the intensity and length of the experimental treatment applied.

5. Treatment with colchicine at a 1 : 1000 dilution, applied during 260 hours, failed to inhibit the regeneration of limbs in tadpoles of *Rana temporaria*, when amputated at the I stage.

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ZOOLOGY

COMPARATIVE DATA ON THE DEVELOPMENT OF REPRODUCTIVE ORGANS IN *EURYGASTER INTEGRICEPS* PUT. IN THE PLAINS OF THE SOUTH OF THE USSR AND IN UZBEKISTAN

By M. J. TEPLYAKOVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 22.IX 1944)

The extreme density of the population of *Eurygaster integriceps* in the South of the European part of the USSR and in Central Asia points to a high degree of adaptation to varying conditions of the surrounding medium: it is largely dependent on the development and function of the imago sex glands which grow intensely since the larval period of life. The complete development of the sex glands depends, however, aside from the endocrine and some other internal factors, also upon the climatic conditions, determining the specific regularity and cyclicity of their development.

The larval development of *Eurygaster integriceps* (which proceeds since the middle of May till the middle of July, and since the middle of April till the beginning of June, in the European part of the USSR and Uzbekistan, respectively) is connected with the formation and growth of the organs of the female and male sex systems and the beginning of oogenesis and spermatogenesis.

Alongside of the continuing morphogenesis, the sex glands in the imagines pass through the latest stages of development, interrupted by the winter diapause. The maturation of sex cells is complete towards the spring of the next year.

The study of the material from the Kashka-Darya region (Uzbekistan), gathered by the expedition of the Academy of Sciences of the USSR in 1942, has revealed certain differences in the development of the sex system of *Eurygaster integriseps* as compared to the European variety of the species under consideration.

The sex system in the new-born male and female larvae in the Uzbekistan race is much similar to that in Rostov and Krasnodar varieties. The female and male gonads consist on each side of the body of 7 disposed fanwise ovary tubes or 7 follicles, respectively. The former show a fan-like arrangement, the latter form bean-like testis. Both are located between the first and second abdominal segments, just like in the European variety, except that there they are more friable and of less perfect shape. As the development proceeds, the differences in the formation of the female and male gonads become more pronounced.

Up to the fourth instar the gonads of the Uzbekistan race are noticeably lagging behind in their development. During the next period the rate of the growth of ovaries becomes somewhat higher, but their differentiation and, therefore, the formation of external characters is retarded, as compared to the European variety, by approximately one instar. The division of oogonia begins probably at the fourth instar, but even at the fifth instar rather long ovary tubules are present (Fig. 1, *A*) which still keep their original disjoined condition, and their tissues are rather loose. Later on the tops of the tubules are gradually coming into contact, and the ovaries

formed become dolioform, in contradistinction to the digitate ovaries of the European variety (Fig. 1, *Б*). At the proximal ends of the tubules of the European race intense oogonial divisions take place by that time giving rise to the first oocytes, part of which is gradually lagging behind in growth to form ultimately the so-called nutrient cells.

Like in the European strain, the male gonads of the larvae of the Usbekistan strain develop in the same way as the female gonads. In the former the division of spermatogonia begins at the third instar; it takes place in the salient follicles of the testes, while the common deferent chamber is gradually formed below the follicles. Since the fourth instar the testes of larvae in the Uzbekistan strain are highly variable in size, being half the size of the testes in the larvae from the Rostov district. Their follicles, however, are well shaped. This stage is probably the beginning of cell divisions in the embryo epithelium. Among the testes of the fifth instar varying in size (Fig. 1, *В*), gonads can be found that are larger in size

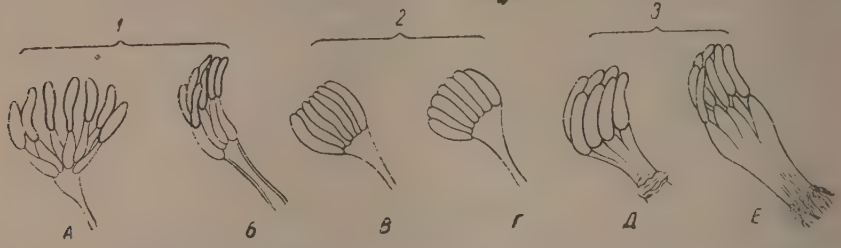

Fig. 1. *1*, left ovaries of the fifth instar larvae of *Eurygaster integriceps*: *А*—Uzbekistan, *Б*—European variety; *2*, left testes of the same age: *В*—Uzbekistan, *Г*—European variety; *3*, left ovaries of imagines before autumnal migration: *Д*—Uzbekistan, *Е*—European variety. All the drawings in Fig. 1 made with the same magnification

as compared to those of the European strain, though their differentiation is apparently retarded. At the beginning of the fifth instar, alongside of growth, the fan-like follicles become spitulate in shape (Fig. 1, *В*, *Г*). In the proximal zones of the follicles of the European bugs of a similar age spermatocytes are present while their division takes place in the distal (lower) ends of the follicles; it also proceeds during the pre-imaginal period. The deferential parts of the follicles are empty at this stage.

The deferent ducts in the larvae of the first instar are very thin, thread-like, without noticeable lumen. In males they have ampulla-like swellings at the end of the IX segment. In females they end in the folds of the future vagina in the VIII segment. Since the third instar the deferent ducts become tubular. The oviducts gradually become thick, flatt and short, uniting into an unpaired oviduct. Below the latter a large vagina is developed as early as at the fifth instar, having a chitinous furrow, close to which a spermatheca and copulative bursae are developed.

Unlike the European variety, the oviducts and the complementary organs here described develop in the Uzbekistan strain more slowly at the beginning; the rate of development becomes normal at the imaginal stage only.

In larvae of European strain the ectodermal pouch-like fold is coming into contact with the ampulla-like swellings at the ends of the seminal ducts since the end of the third instar. Inside this fold develops afterwards the seminal duct with its upper part extended like a reservoir. During the pre-imaginal period, along with the growth of the testes and intense division of spermatocytes, proceeds the shortening of seminal ducts and begins the development of accessory glands. In larvae of the Uzbekistan variety these processes go on somewhat slower.

Previous to migration from the field (July and June, for the European part of the USSR and Uzbekistan, respectively) the development of the reproductive system in females is in general complete (Fig. 1, *Е*). The lengthened and thickened ovary tubules are often sausage-like bodies, clearly separated from their deferent parts. The proximal ends of the ovary tubules of telotrophic type contain growing

eggs and nutrient cells. The protoplasmic strands of the latters unite later with the oocytes of the next chambers. A large amount of tracheal tubules develop around the ovary tubes. By that time at the end of the broad and short oviducts accessory fringed glands have developed, producing a sticky shell of the eggs, when these pass through the unpaired oviduct. The duct of the rounded spermatheca opens at the level of the upper third into the chitinous vaginal furrow. The burses are located on the sides of the vagina. Below the latter are supporting plates. Unlike the European strain, in the females of the Kashka-Darya variety the ovary tubes at the period of migration are considerably thinner and shorter, which is due to a large number of tracheae developed (Fig. 1, *Д*). The accessory oviduct glands are somewhat less developed, but as compared to the larval period the development of the whole reproductive system is much more advanced.

The two varieties under comparison migrate from the field having underdeveloped ovaries, as described by Sokolov ([2]) for *Eurygaster maura*, and by many others for *Hemiptera*. This stage of development of the ovaries, or a somewhat more advanced one, is characteristic of females from $9^1/_2$ to 11 months, according to the conditions of the beginning of aestivation in the plain, piedmonts or mountains of Central Asia. The stage of intense growth of the oocytes begins in the European plain early in spring at the expense of alimentary material accumulated. The maturation of the ovaries proceeds under condition of intense feeding in the spring. The second period of the growth of oocytes and the beginning of intense accumulation of yolk depends upon the altitude and conditions prevailing at the process of migration, and may last till the end of April. The migration from the mountains to the plain ([1]) also proceeds in successive waves since the 20th of March till the 20th of May.

Owing to general growth and the numerous trachea passing under their membrane, the testes of the imagines before migration are much increased as compared to those of the larval period of development. The swelling of the testes is especially conspicuous in males of the Uzbekistan variety; reducing their relative body weight, it facilitates their flight to the mountains. The 5 annexes and one rudimentary gland have at this period the shape of sinuous balls near the opening of the seminal ducts into the extended part of *vasa efferentia*. In the males of Uzbekistan variety the annex glands at this period are underdeveloped and less in number. Intense division of spermatocyte and formation of prespermatides takes place in the follicles.

Both males and females migrate with underdeveloped sex glands. Later on, the testes enter the maturation period. They become piriform, the red pigment appears in their membrane, prespermatides are transformed into spermatozoa; granular secretion is produced in the increased and thickened annexes. Spermatozoa are formed even before spring migration. They fill the follicles, discharging chamber, and seminal ducts. The swollen accessory glands are filled with secretion. The beginning of maturation of testes and, therefore, their final formation, as well as the formation of the seminal ducts and accumulation of the pigment in the latter, is somewhat retarded. The development of the different sex organs in the males proceeds gradually, in accordance with their functioning, similar to that of the females.

From the morpho-physiological point of view the development of gonads in *Eurygaster integriceps* is characterized by the presence of a period of growth connected with the proliferation of the germinating cells and the beginning of the growth of the spermatocytes and oocytes, up to the summer migration, and the second, longer period of maturation that continues till the $9^1/_2$th month, or even more in alpine districts. At this second period the testes undergo a decrease in size caused by the condensation of the maturating germ cells. The testes assume their definitive form. In connexion with the growth of oocytes and accumulation of a considerable amount of yolk the ovary tubes increase in size and becom bead-like in appearance.

The direct proportionality between the growths rate of *corpora allata* (analogous to the pituitary body in vertebrates) and the rate of development of the gonads

described for the European strain of *Eurygaster integriceps* (3) was found to be true for the Uzbek variety also. It must be pointed out that there is a definite relation between the rate of growth of *corpora allata* and the altitude of hibernation; a general trend in the seasonal influence shown in the Fig. 2 is, however, evident.

In connexion with the anatomical and physiological differences above-described the biology of the Uzbekistan strain of the species under consideration is also more complicated. Owing to the difference in the conditions of wintering and the rates of maturation of the sex gonads, spring migration to the valley does not proceed simultaneously, leading to different concentration of the pest in the

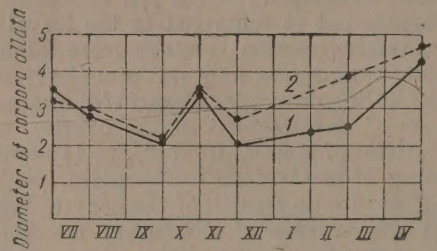

Fig. 2. Along the ordinates is plotted in relative units the mean arithmetical between large and small diameters of *corpora allata* (mostly of an irregular round shape). *1*, males; *2*, females

fields. As sun radiation becomes more intense, the average day temperature accelarates the development of the eggs and larvae, reducing in this way the mortality in population. On the other hand, high temperature and low moisture have a detrimental effect upon the larvae of the first and second instars, thus decreasing the population density. Both factors are levelling out the differences in the time of development of the larvae, bringing into equilibrium the total number of individuals in the population, and leading to an apparently synchronous development of a new generation.

The differences in the conditions of eastivation, hibernation and reservation bring about a different rate of development of gonads and, therefore, extensive variation in their size. In the subsequent climatic factors bring about an undulatory character of the spring migration, a mixing of the individuals migrating from the different altitudes.

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